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M-THEORY
(THE THEORY FORMERLY KNOWN AS STRINGS)[1]

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ABSTRACT

Superunification underwent a major paradigm shift in 1984 when eleven-dimensional supergravity was knocked off its pedestal by ten-dimensional superstrings. This last year has witnessed a new shift of equal proportions: perturbative ten-dimensional superstrings have in their turn been superseded by a new non-perturbative theory called *M-theory*, which describes supermembranes and superfivebranes, which subsumes all five consistent string theories and whose low energy limit is, ironically, eleven-dimensional supergravity. In particular, six-dimensional string/string duality follows from membrane/fivebrane duality by compactifying M-theory on $S^1/Z_2 \times K3$ (heterotic/heterotic duality) or $S^1 \times K3$ (Type IIA/heterotic duality) or $S^1/Z_2 \times T^4$ (heterotic/Type IIA duality) or $S^1 \times T^4$ (Type IIA/Type IIA duality).

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1 Ten to eleven: it is not too late

The maximum spacetime dimension in which one can formulate a consisistent supersymmetric theory is eleven[3]. For this reason in the early 1980's many physicists looked to $D = 11$ supergravity [2], in the hope that it might provide that superunification [3] they were all looking for. Then in 1984 superunification underwent a major paradigm shift: eleven-dimensional supergravity was knocked off its pedestal by ten-dimensional superstrings [4], and eleven dimensions fell out of favor. This last year, however, has witnessed a new shift of equal proportions: perturbative ten-dimensional superstrings have in their turn been superseded by a new non-perturbative theory called *M-theory*, which describes (amongst other things) supersymmetric extended objects with two spatial dimensions (*supermembranes*), and five spatial dimensions (*superfivebranes*), which subsumes all five consistent string theories and whose low energy limit is, ironically, eleven-dimensional supergravity.

The reason for this reversal of fortune of eleven dimensions is due, in large part, to the 1995 paper by Witten [5]. One of the biggest problems with $D = 10$ string theory [4] is that there are *five* consistent string theories: Type I $SO(32)$, heterotic $SO(32)$, heterotic $E_8 \times E_8$, Type IIA and Type IIB. As a candidate for a unique *theory of everything*, this is clearly an embarrassment of riches. Witten put forward a convincing case that this distinction is just an artifact of perturbation theory and that non-perturbatively these five theories are, in fact, just different corners of a deeper theory. Moreover, this deeper theory, subsequently dubbed *M-theory*, has $D = 11$ supergravity as its low energy limit! Thus the five string theories and $D = 11$ supergravity represent six different special points[4] in the moduli space of M-theory. The small parameters of perturbative string theory are provided by $< e^{\Phi} >$, where Φ is the dilaton field, and $< e^{\sigma_i} >$ where σ_i are the moduli fields which arise after compactification. What makes M-theory at once intriguing and yet difficult to analyse is that in $D = 11$ there is neither dilaton nor moduli and hence the theory is intrinsically non-perturbative. Consequently, the ultimate meaning of M-theory is still unclear, and Witten has suggested

[3]The field-theoretic reason is based on the prejudice that there be no massless particles with spins greater than two [1]. However, as discussed in section (5), $D = 11$ emerges naturally as the maximum dimension admitting super p-branes in Minkowski signature.

[4]Some authors take the phrase *M-theory* to refer merely to this sixth corner of the moduli space. With this definition, of course, M-theory is no more fundamental than the other five corners. For us, *M-theory* means the whole kit and caboodle.

that in the meantime, M should stand for "Magic", "Mystery" or "Membrane", according to taste.

The relation between the membrane and the fivebrane in $D = 11$ is analogous to the relation between electric and magnetic charges in $D = 4$. In fact this is more than an analogy: electric/magnetic duality in $D = 4$ string theory [6, 7] follows as a consequence of string/string duality in $D = 6$ [8]. The main purpose of the present paper is to show how $D = 6$ string/string duality [9, 10, 11, 12, 13, 14, 5] follows, in its turn, as a consequence of membrane/fivebrane duality in $D = 11$. In particular, heterotic/heterotic duality, Type IIA/heterotic duality, heterotic/Type IIA duality and Type IIA/Type IIA duality follow from membrane/fivebrane duality by compactifying M-theory on $S^1/Z_2 \times K3$ [15], $S^1 \times K3$ [16], $S^1/Z_2 \times T^4$ and $S^1 \times T^4$, respectively.

First, however, I want to pose the question: "Should we have been surprised by the eleven-dimensional origin of string theory?"

2 Type II A&M theory

The importance of eleven dimensions is no doubt surprising from the point of view of perturbative string theory; from the point of view of membrane theory, however, there were already tantalizing hints in this direction:

(i) **K3 compactification**

The four-dimensional compact manifold $K3$ plays a ubiquitous role in much of present day M-theory. It was first introduced as a compactifying manifold in 1983 [18] when it was realised that the number of unbroken supersymmetries surviving compactification in a Kaluza-Klein theory depends on the *holonomy* group of the extra dimensions. By virtue of its $SU(2)$ holonomy, $K3$ preserves precisely half of the supersymmetry. This means, in particular, that an $N = 2$ theory on $K3$ has the same number of supersymmetries as an $N = 1$ theory on T^4, a result which was subsequently to prove of vital importance for string/string duality. In 1986, it was pointed out [19] that $D = 11$ supergravity on $R^{10-n} \times K3 \times T^{n-3}$ [18] and the $D = 10$ heterotic string on $R^{10-n} \times T^n$ [20] not only have the same supersymmetry but also the same moduli spaces of vacua, namely

$$\mathcal{M} = \frac{SO(16+n,n)}{SO(16+n) \times SO(n)} \tag{2.1}$$

It took almost a decade for this "coincidence" to be explained but we now know that M-theory on $R^{10-n} \times K3 \times T^{n-3}$ is dual to the heterotic string on $R^{10-n} \times T^n$.

(ii) **Superstrings in D=10 from supermembranes in D=11**

Eleven dimensions received a big shot in the arm in 1987 when the $D = 11$ supermembrane was discovered [21]. The bosonic sector of its $d = 3$ worldvolume Green-Schwarz action is given by:

$$\begin{aligned} S_3 &= T_3 \int d^3\xi \Big[-\frac{1}{2}\sqrt{-\gamma}\gamma^{ij}\partial_i X^M \partial_j X^N G_{MN}(X) + \frac{1}{2}\sqrt{-\gamma} \\ &\qquad - \frac{1}{3!}\epsilon^{ijk}\partial_i X^M \partial_j X^N \partial_k X^P C_{MNP}(X) \Big] \end{aligned} \tag{2.2}$$

where T_3 is the membrane tension, ξ^i ($i = 1, 2, 3$) are the worldvolume coordinates, γ^{ij} is the worldvolume metric and $X^M(\xi)$ are the spacetime coordinates ($M = 0, 1, \ldots, 10$). Kappa symmetry [21] then demands that the background metric G_{MN} and background 3-form potential C_{MNP} obey the classical field equations of $D = 11$ supergravity [2], whose bosonic action is

$$I_{11} = \frac{1}{2\kappa_{11}^2} \int d^{11}x \sqrt{-G} \left[R_G - \frac{1}{2 \cdot 4!} {K_4}^2 \right] - \frac{1}{12\kappa_{11}^2} \int C_3 \wedge K_4 \wedge K_4 \tag{2.3}$$

where $K_4 = dC_3$ is the 4-form field strength. In particular, K_4 obeys the field equation

$$d * K_4 = -\frac{1}{2}{K_4}^2 \tag{2.4}$$

and the Bianchi identity

$$dK_4 = 0 \tag{2.5}$$

It was then pointed out [22] that in an $R^{10} \times S^1$ topology the weakly coupled ($d = 2, D = 10$) Type IIA superstring follows by wrapping the ($d = 3, D = 11$) supermembrane around the circle in the limit that its radius R shrinks to zero. In particular, the Green-Schwarz action of the string follows in this way from the Green-Schwarz action of the membrane. It was necessary to take this $R \to 0$ limit in order to send to infinity the masses of the (at the time) unwanted Kaluza-Klein modes which had no place in weakly coupled Type IIA theory. The $D = 10$ dilaton, which governs the strength of the string coupling, is just a component of the $D = 11$ metric.

A critique of superstring orthodoxy *circa* 1987, and its failure to accommodate the eleven-dimensional supermembrane, may be found in [23].

(iii) **U-duality (when it was still non-U)**

Based on considerations of this $D = 11$ supermembrane, which on further compactification treats the dilaton and moduli fields on the same footing, it was conjectured [26] in 1990 that discrete subgroups of all the old non-compact global symmetries of compactified supergravity [24, 25] (e.g $SL(2, R)$, $O(6, 6)$, E_7) should be promoted to duality symmetries of the supermembrane. Via the above wrapping around S^1, therefore, they should be also be inherited by the Type IIA string [26].

(iv) **D=11 membrane/fivebrane duality**

In 1991, the supermembrane was recovered as an elementary solution of $D = 11$ supergravity which preserves half of the spacetime supersymmetry [27]. Making the three/eight split $X^M = (x^\mu, y^m)$ where $\mu = 0, 1, 2$ and $m = 3, ..., 10$, the metric is given by

$$ds^2 = (1 + k_3/y^6)^{-2/3} dx^\mu dx_\mu + (1 + k_3/y^6)^{1/3}(dy^2 + y^2 d{\Omega_7}^2) \tag{2.6}$$

and the four-form field strength by

$$\tilde{K}_7 \equiv *K_4 = 6k_3\epsilon_7 \tag{2.7}$$

where the constant k_3 is given by

$$k_3 = \frac{2{\kappa_{11}}^2 T_3}{\Omega_7} \tag{2.8}$$

Here ϵ_7 is the volume form on S^7 and Ω_7 is the volume. The mass per unit area of the membrane $\mathcal{M}_3$ is equal to its tension:

$$\mathcal{M}_3 = T_3 \tag{2.9}$$

This *elementary* solution is a singular solution of the supergravity equations coupled to a supermembrane source and carries a Noether "electric" charge

$$Q = \frac{1}{\sqrt{2}\kappa_{11}} \int_{S^7} (*K_4 + C_3 \wedge K_4) = \sqrt{2}\kappa_{11} T_3 \tag{2.10}$$

Hence the solution saturates the Bogomol'nyi bound $\sqrt{2}\kappa_{11}\mathcal{M}_3 \geq Q$. This is a consequence of the preservation of half the supersymmetries which is also intimately linked with the worldvolume kappa symmetry. The zero modes of this solution belong to a $(d = 3, n = 8)$ supermultiplet consisting of eight scalars and eight spinors (ϕ^I, χ^I), with $I = 1, ..., 8$, which correspond to the eight Goldstone bosons and their superpartners associated with breaking of the eight translations transverse to the membrane worldvolume.

In 1992, the superfivebrane was discovered as a soliton solution of $D = 11$ supergravity also preserving half the spacetime supersymmetry [28]. Making the six/five split $X^M = (x^\mu, y^m)$ where $\mu = 0, 1, 2, 3, 4, 5$ and $m = 6, ..., 10$, the metric is given by

$$ds^2 = (1 + k_6/y^3)^{-1/3} dx^\mu dx_\mu + (1 + k_6/y^3)^{2/3}(dy^2 + y^2 d\Omega_4{}^2) \tag{2.11}$$

and the four-index field-strength by

$$K_4 = 3k_6\epsilon_4 \tag{2.12}$$

where the fivebrane tension $\tilde{T}_6$ is related to the constant k_6 by

$$k_6 = \frac{2\kappa_{11}{}^2\tilde{T}_6}{3\Omega_4} \tag{2.13}$$

Here ϵ_4 is the volume form on S^4 and Ω_4 is the volume. The mass per unit 5-volume of the fivebrane $\mathcal{M}_6$ is equal to its tension:

$$\mathcal{M}_6 = \tilde{T}_6 \tag{2.14}$$

This *solitonic* solution is a non-singular solution of the source-free equations and carries a topological "magnetic" charge

$$P = \frac{1}{\sqrt{2}\kappa_{11}} \int_{S^4} K_4 = \sqrt{2}\kappa_{11}\tilde{T}_6 \tag{2.15}$$

Hence the solution saturates the Bogomol'nyi bound $\sqrt{2}\kappa_{11}\mathcal{M}_6 \geq P$. Once again, this is a consequence of the preservation of half the supersymmetries. The covariant action for this $D = 11$ superfivebrane is still unknown (see [29, 30] for recent progress) but consideration of the soliton zero modes [31, 14, 32] means that the gauged fixed action must be described by the same chiral antisymmetric tensor multiplet $(B^-{}_{\mu\nu}, \lambda^I, \phi^{[IJ]})$ as that of the Type IIA fivebrane [33, 34]. Note that in addition to the five scalars corresponding to the five translational Goldstone bosons, there is also a 2-form $B^-{}_{\mu\nu}$ whose 3-form field strength is anti-self dual and which describes three degrees of freedom.

The electric and magnetic charges obey a Dirac quantization rule [35, 36]

$$QP = 2\pi n \qquad n = \text{integer} \tag{2.16}$$

Or, in terms of the tensions [37, 11],

$$2\kappa_{11}{}^2 T_3 \tilde{T}_6 = 2\pi n \tag{2.17}$$

This naturally suggests a $D=11$ membrane/fivebrane duality. Note that this reduces the three dimensionful parameters T_3, $\tilde{T}_6$ and κ_{11} to two. Moreover, it was recently shown [16] that they are not independent. To see this, we note from (2.2) that C_3 has period $2\pi/T_3$ so that K_4 is quantized according to

$$\int K_4 = \frac{2\pi n}{T_3} \quad n = integer \tag{2.18}$$

Consistency of such C_3 periods with the spacetime action, (2.3), gives the relation[5]

$$\frac{(2\pi)^2}{{\kappa_{11}}^2 T_3^3} \in 2Z \tag{2.19}$$

From (2.17), this may also be written as

$$2\pi \frac{\tilde{T}_6}{{T_3}^2} \in Z \tag{2.20}$$

Thus the tension of the singly charged fivebrane is given by

$$\tilde{T}_6 = \frac{1}{2\pi} {T_3}^2 \tag{2.21}$$

(*v*) **Hidden eleventh dimension**

We have seen how the $D=10$ Type IIA string follows from $D=11$. Is it possible to go the other way and discover an eleventh dimension hiding in $D=10$? In 1993, it was recognized [40] that by dualizing a vector into a scalar on the gauge-fixed $d=3$ worldvolume of the Type IIA supermembrane, one increases the number of worldvolume scalars (i.e transverse dimensions) from 7 to 8 and hence obtains the corresponding worldvolume action of the $D=11$ supermembrane. Thus the $D=10$ Type IIA theory contains a hidden $D=11$ Lorentz invariance! This device was subseqently used [41, 42] to demonstrate the equivalence of the actions of the $D=10$ Type IIA membrane and the Dirichlet twobrane [43].

(*vi*) **U-duality**

Of the conjectured Cremmer-Julia symmetries referred to in (*iii*) above, the case for a target space $O(6,6;Z)$ (T-*duality*) in perturbative string theory had already been made, of course [44]. Stronger evidence for an $SL(2,Z)$ (S-*duality*) in string theory was subsequently provided in [6, 7] where it was pointed out that it corresponds to a *non-perturbative* electric/magnetic symmetry.

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[5] This corrects a factor of two error in [16] and brings us into agreement with a subsequent D-brane derivation [38] of (2.21). I am grateful to Shanta De Alwis [39] for pointing out the source of the error.

In 1994, stronger evidence for the combination of S and T into a discrete duality of Type II strings, such as $E_7(Z)$ in $D = 4$, was provided in [13], where it was dubbed *U-duality*. Moreover, the BPS spectrum necessary for this U-duality was given an explanation in terms of the wrapping of either the $D = 11$ membrane or $D = 11$ fivebrane around the extra dimensions. This paper also conjectured a non-perturbative $SL(2, Z)$ of the Type IIB string in $D = 10$.

(vii) **Black Holes**

In 1995, it was conjectured [32] that the $D = 10$ Type IIA superstring should be identified with the $D = 11$ supermembrane compactified on S^1, even for large R. The $D = 11$ Kaluza-Klein modes (which, as discussed in (ii) above, had no place in the *perturbative* Type IIA theory) were interpreted as charged extreme black holes of the Type IIA theory.

$(viii)$ **D=11 membrane/fivebrane duality and anomalies**

Membrane/fivebrane duality interchanges the roles of field equations and Bianchi identities. From (2.4), the fivebrane Bianchi identity reads

$$d\tilde{K}_7 = -\frac{1}{2}{K_4}^2 \tag{2.22}$$

However, it was recognized in 1995 that such a Bianchi identity will in general require gravitational Chern-Simons corrections arising from a sigma-model anomaly on the fivebrane worldvolume [16]

$$d\tilde{K}_7 = -\frac{1}{2}K_4^2 + \frac{2\pi}{\tilde{T}_6}\tilde{X}_8 \tag{2.23}$$

where the 8-form polynomial $\tilde{X}_8$, quartic in the gravitational curvature R, describes the Lorentz $d = 6$ worldvolume anomaly of the $D = 11$ fivebrane. Although the covariant fivebrane action is unknown, we know that the gauge fixed theory is described by the chiral antisymmetric tensor multiplet $(B^-_{\mu\nu}, \lambda^I, \phi^{[IJ]})$, and it is a straightforward matter to read off the anomaly polynomial from the literature. See, for example [45]. We find

$$\tilde{X}_8 = \frac{1}{(2\pi)^4}[-\frac{1}{768}(\mathrm{tr}R^2)^2 + \frac{1}{192}\mathrm{tr}R^4] \tag{2.24}$$

Thus membrane/fivebrane duality predicts a spacetime correction to the $D = 11$ supermembrane action [16]

$$I_{11}(Lorentz) = T_3 \int C_3 \wedge \frac{1}{(2\pi)^4}[-\frac{1}{768}(\mathrm{tr}R^2)^2 + \frac{1}{192}\mathrm{tr}R^4] \tag{2.25}$$

Such a correction was also derived in a somewhat different way in [17]. This prediction is intrinsically M-theoretic, with no counterpart in ordinary $D = 11$ supergravity. However, by simultaneous dimensional reduction [22] of $(d = 3, D = 11)$ to $(d = 2, D = 10)$ on S^1, it translates into a corresponding prediction for the Type IIA string:

$$I_{10}(Lorentz) = T_2 \int B_2 \wedge \frac{1}{(2\pi)^4}[-\frac{1}{768}(\mathrm{tr}R^2)^2 + \frac{1}{192}\mathrm{tr}R^4] \tag{2.26}$$

where B_2 is the string 2-form and $T_2 = 1/2\pi\alpha'$ is the string tension.

As a consistency check we can compare this prediction with previous results found by explicit string one-loop calculations. These have been done in two ways: either by computing directly in $D = 10$ the Type IIA anomaly polynomial [46] following [47], or by compactifying to $D = 2$ on an 8-manifold M and computing the B_2 one-point function [48]. We indeed find agreement. Thus using $D = 11$ membrane/fivebrane duality we have correctly reproduced the corrections to the B_2 field equations of the $D = 10$ Type IIA string (a mixture of tree-level and string one-loop effects) starting from the Chern-Simons corrections to the Bianchi identities of the $D = 11$ superfivebrane (a purely tree-level effect). It would be interesting to know, on the membrane side, what calculation in $D = 11$ M-theory, when reduced on S^1, corresponds to this one-loop Type IIA string amplitude calculation in $D = 10$. Understanding this may well throw a good deal of light on the mystery of what M-theory really is!

(ix) **Heterotic string from fivebrane wrapped around $K3$**

In 1995 it was shown that, when wrapped around $K3$ with its 19 self-dual and 3 anti-self-dual 2-forms, the $d = 6$ worldvolume fields of the $D = 11$ fivebrane (or Type IIA fivebrane) $({B^-}_{\mu\nu}, \lambda^I, \phi^{[IJ]})$ reduce to the $d = 2$ worldsheet fields of the heterotic string in $D = 7$ (or $D = 6$) [49, 50]. The 2-form yields $(19, 3)$ left and right moving bosons, the spinors yield $(0, 8)$ fermions and the scalars yield $(5, 5)$ which add up to the correct worldsheet degrees of freedom of the heterotic string [49, 50].

A consistency check is provided [16] by the derivation of the Yang-Mills and Lorentz Chern-Simons corrections to the Bianchi identity of the heterotic string starting from the fivebrane Bianchi identity given in $(viii)$. Making the seven/four split $X^M = (x^\mu, y^m)$ where $\mu = 0, ..., 6$ and $m = 7, 8, 9, 10$, the original set of $D = 11$ fields may be decomposed in a basis of harmonic p-forms on $K3$. In particular, we expand C_3 as

$$C_3(X) = C_3(x) + \frac{1}{2T_3} \sum C_1^I(x)\omega_2^I(y) \tag{2.27}$$

where ω_2^I, $I = 1, \ldots, 22$ are an integral basis of b_2 harmonic two-forms on $K3$. Following [12], let us define the dual string 3-form $\tilde{H}_3$ by

$$T_2 \tilde{H}_3 = \tilde{T}_6 \int_{K3} \tilde{K}_7, \tag{2.28}$$

The dual string Lorentz anomaly polynomial, $\tilde{X}_4$, is given by

$$\tilde{X}_4 = \int_{K3} \tilde{X}_8 \quad = \quad \frac{1}{(2\pi)^2} \frac{1}{192} \mathrm{tr} R^2 p_1(K3) \tag{2.29}$$

where $p_1(K3)$ is the Pontryagin number of $K3$

$$p_1(K3) = -\frac{1}{8\pi^2} \int_{K3} \mathrm{tr} R_0^2 = -48 \tag{2.30}$$

We may now integrate (2.23) over $K3$, using (2.21) to find

$$d\tilde{H}_3 = -\frac{\alpha'}{4} [K_2^I K_2^J d_{IJ} + \mathrm{tr} R^2] \tag{2.31}$$

where $K_2^I = dC_1^I$ and where d_{IJ} is the intersection matrix on $K3$, given by

$$d_{IJ} = \int_{K3} \omega_2^I \wedge \omega_2^J \tag{2.32}$$

which has $b_2^+ = 3$ positive and $b_2^- = 19$ negative eigenvalues. Thus we see that this form of the Bianchi identity corresponds to a $D = 7$ toroidal compactification of a heterotic string at a generic point on the Narain lattice [20]. Thus we have reproduced the $D = 7$ Bianchi identity of the heterotic string, starting from the $D = 11$ fivebrane.

For use in section (3), we note that if we replace $K3$ by T^4 in the above derivation, the 2-form now yields $(3, 3)$ left and right moving bosons, the spinors now yield $(8, 8)$ fermions and the scalars again yield $(5, 5)$ which add up to the correct worldsheet degrees of freedom of the Type IIA string. In this case, the Bianchi identity becomes $d\tilde{H}_3 = 0$ as it should be.

(x) **N=1 in D=4**

Also in 1995 it was noted [51, 52, 53, 55, 54, 56, 64] that $N = 1$ heterotic strings can be dual to $D = 11$ supergravity compactified on seven-dimensional spaces of G_2 holonomy which also yield $N = 1$ in $D = 4$ [57].

(xi) **Non-perturbative effects**

Also in 1995 it was shown [58] that membranes and fivebranes of the Type IIA theory, obtained by compactification on S^1, yield e^{-1/g_s} effects, where g_s is the string coupling.

(*xii*) **SL(2,Z)**

Also in 1995, strong evidence was provided for identifying the Type IIB string on $R^9 \times S^1$ with M-theory on $R^9 \times T^2$ [38, 64]. In particular, the conjectured $SL(2,Z)$ of the Type IIB theory discussed in (vi) above is just the modular group of the M-theory torus[6]

(*xiii*) $\mathrm{E}_8 \times \mathrm{E}_8$ **heterotic string**

Also in 1995 (that *annus mirabilis!*), strong evidence was provided for identifying the $E_8 \times E_8$ heterotic string [7] on R^{10} with M-theory on $R^{10} \times S^1/Z_2$ [67].

This completes our summary of M-theory before M-theory was cool. The *phrase* M-theory (though, as I hope to have shown, not the *physics* of M-theory) first made its appearance in October 1995 [38, 67]. This was also the month that it was proposed [43] that the Type II p-branes carrying Ramond-Ramond charges can be given an exact conformal field theory description via open strings with Dirichlet boundary conditions, thus heralding the era of $D-branes$. Since then, evidence in favor of M-theory and D-branes has been appearing daily on the internet, including applications to black holes [62], length scales shorter than the string scale [59] and even phenomenology [60, 61]. We refer the reader to the review by Schwarz [63] for these more recent developments in M-theory, to the review by Polchinski [69] for developments in D-branes and to the paper by Aharony, Sonnenschein and Yankielowicz [70] for the connection between the two (since D-branes are intrinsically ten-dimensional and M-theory is eleven-dimensional, this is not at all obvious). Here, we wish to focus on a specific application of M-theory, namely the derivation of string/string dualities.

3 String/string duality from M-theory

Let us consider M-theory, with its fundamental membrane and solitonic fivebrane, on $R^6 \times M_1 \times \tilde{M}_4$ where M_1 is a one-dimensional compact space of radius R and $\tilde{M}_4$ is a four-

[6]Two alternative explanations of this $SL(2,Z)$ had previously been given: (a) identifying it with the S-duality [16] of the $d=4$ Born-Infeld worldvolume theory of the self-dual Type IIB superthreebrane [65], and (b) using the four-dimensional heterotic/Type IIA/Type IIB triality [66] by noting that this $SL(2,Z)$, while non-perturbative for the Type IIB string, is perturbative for the heterotic string.

[7]It is ironic that, having hammered the final nail in the coffin of $D=11$ supergravity by telling us that it can never yield a *chiral* theory when compactified on a manifold [68], Witten pulls it out again by telling us that it does yield a chiral theory when compactified on something that is not a manifold!

dimensional compact space of volume V. We may obtain a fundamental string on R^6 by wrapping the membrane around M_1 and reducing on $\tilde{M}_4$. Let us denote fundamental string sigma-model metrics in $D = 10$ and $D = 6$ by G_{10} and G_6. Then from the corresponding Einstein Lagrangians

$$\sqrt{-G_{11}}R_{11} = R^{-3}\sqrt{-G_{10}}R_{10} = \frac{V}{R}\sqrt{-G_6}R_6 \tag{3.1}$$

we may read off the strength of the string couplings in $D = 10$ [15]

$$\lambda_{10}{}^2 = R^3 \tag{3.2}$$

and $D = 6$

$$\lambda_6{}^2 = \frac{R}{V} \tag{3.3}$$

Similarly we may obtain a solitonic string on R^6 by wrapping the fivebrane around $\tilde{M}_4$ and reducing on M_1. Let us denote the solitonic string sigma-model metrics in $D = 7$ and $D = 6$ by $\tilde{G}_7$ and $\tilde{G}_6$. Then from the corresponding Einstein Lagrangians

$$\sqrt{-G_{11}}R_{11} = V^{-3/2}\sqrt{-\tilde{G}_7}\tilde{R}_7 = \frac{R}{V}\sqrt{-\tilde{G}_6}\tilde{R}_6 \tag{3.4}$$

we may read off the strength of the string couplings in $D = 7$ [15]

$$\tilde{\lambda}_7^2 = V^{3/2} \tag{3.5}$$

and $D = 6$

$$\tilde{\lambda}_6^2 = \frac{V}{R} \tag{3.6}$$

Thus we see that the fundamental and solitonic strings are related by a strong/weak coupling:

$$\tilde{\lambda}_6^2 = 1/\lambda_6{}^2 \tag{3.7}$$

We shall be interested in $M_1 = S^1$ (in which case from (ii) of section (2) the fundamental string will be Type IIA) or $M_1 = S^1/Z_2$ (in which case from $(xiii)$ of section (2) the fundamental string will be heterotic $E_8 \times E_8$). Similarly, we will be interested in $\tilde{M}_4 = T^4$ (in which case from (ix) of section (2) the solitonic string will be Type IIA) or $\tilde{M}_4 = K3$ (in which case from (ix) of section (2) the solitonic string will be heterotic). Thus there are four possible scenarios which are summarized in Table 1. (N_+, N_-) denotes the $D = 6$ spacetime supersymmetries. In each case, the fundamental string will be weakly coupled as we shrink

$(\mathbf{N_+}, \mathbf{N_-})$	$\mathbf{M_1}$	$\tilde{\mathbf{M}}_\mathbf{4}$	**fundamental string**	**dual string**
$(1, 0)$	S^1/Z_2	$K3$	*heterotic*	*heterotic*
$(1, 1)$	S^1	$K3$	*Type IIA*	*heterotic*
$(1, 1)$	S^1/Z_2	T^4	*heterotic*	*Type IIA*
$(2, 2)$	S^1	T^4	*Type IIA*	*Type IIA*

Table 1: String/string dualities

the size of the wrapping space M_1 and the dual string will be weakly coupled as we shrink the size of the wrapping space $\tilde{M}_4$.

In fact, there is in general a topological obstruction to wrapping the fivebrane around $\tilde{M}_4$ provided by (2.18) because the fivebrane cannot wrap around a 4-manifold that has $n \neq 0$[8]. This is because the anti-self-dual 3-form field strength T on the worldvolume of the fivebrane obeys [41, 17]

$$dT = K_4 \tag{3.8}$$

and the existence of a solution for T therefore requires that K_4 must be cohomologically trivial. For M-theory on $R^6 \times S^1/Z_2 \times T^4$ this is no problem. For M theory on $R^6 \times S^1/Z_2 \times K3$, with instanton number k in one E_8 and $24 - k$ in the other, however, the flux of K_4 over $K3$ is [15]

$$n = 12 - k \tag{3.9}$$

Consequently, the M-theoretic explanation of heterotic/heterotic duality requires $E_8 \times E_8$ with the symmetric embedding $k = 12$. This has some far-reaching implications. For example, the duality exchanges gauge fields that can be seen in perturbation theory with gauge fields of a non-perturbative origin [15].

The dilaton $\tilde{\Phi}$, the string σ-model metric $\tilde{G}_{MN}$ and 3-form field strength $\tilde{H}$ of the dual string are related to those of the fundamental string, Φ, G_{MN} and H by the replacements [11, 12]

$$\Phi \rightarrow \tilde{\Phi} = -\Phi$$

$$G_{MN} \rightarrow \tilde{G}_{MN} = e^{-\Phi} G_{MN}$$

[8]Actually, as recently shown in [71], the object which must have integral periods is not $T_3 K_4/2\pi$ but rather $T_3 K_4/2\pi - p_1/4$ where p_1 is the first Pontryagin class. This will not affect our conclusions, however.

$$H \to \tilde{H} = e^{-\Phi} * H \tag{3.10}$$

In the case of heterotic/Type IIA duality and Type IIA/heterotic duality, this operation takes us from one string to the other, but in the case of heterotic/heterotic duality and Type IIA/Type IIA duality this operation is a discrete symmetry of the theory. This Type IIA/Type IIA duality is discussed in [78] and we recognize this symmetry as subgroup of the $SO(5,5;Z)$ U-duality [26, 13, 79] of the $D=6$ Type IIA string.

Vacua with $(N_+, N_-) = (1,0)$ in $D=6$ have been the subject of much interest lately. In addition to DMW vacua [16] discussed above, obtained from M-theory on $S^1/Z_2 \times K3$, there are also the GP vacua [72, 73, 74] obtained from the $SO(32)$ theory on $K3$ and the MV vacua [85, 86] obtained from *F-theory* [75] on Calabi-Yau. Indeed, all three categories are related by duality [80, 85, 83, 97, 84, 76, 86, 81]. In particular, the DMW heterotic strong/weak coupling duality gets mapped to a T-duality of the Type I version of the $SO(32)$ theory, and the non-perturbative gauge symmetries of the DMW model arise from small $Spin(32)/Z_2$ instantons in the heterotic version of the $SO(32)$ theory [76]. Because heterotic/heterotic duality interchanges worldsheet and spacetime loop expansions – or because it acts by duality on H – the duality exchanges the tree level Chern-Simons contributions to the Bianchi identity

$$\begin{aligned} dH &= \alpha'(2\pi)^2 X_4 \\ X_4 &= \frac{1}{4(2\pi)^2}[\mathrm{tr}R^2 - \Sigma_\alpha v_\alpha \mathrm{tr}F_\alpha{}^2] \end{aligned} \tag{3.11}$$

with the one-loop Green-Schwarz corrections to the field equations

$$\begin{aligned} d\tilde{H} &= \alpha'(2\pi)^2 \tilde{X}_4 \\ \tilde{X}_4 &= \frac{1}{4(2\pi)^2}[\mathrm{tr}R^2 - \Sigma_\alpha \tilde{v}_\alpha \mathrm{tr}F_\alpha{}^2] \end{aligned} \tag{3.12}$$

Here F_α is the field strength of the α^{th} component of the gauge group, tr denotes the trace in the fundamental representation, and $v_\alpha, \tilde{v}_\alpha$ are constants. In fact, the Green-Schwarz anomaly cancellation mechanism in six dimensions requires that the anomaly eight-form I_8 factorize as a product of four-forms,

$$I_8 = X_4 \tilde{X}_4, \tag{3.13}$$

and a six-dimensional string-string duality with the general features summarized above would exchange the two factors [12]. Moreover, supersymmetry relates the coefficients $v_\alpha, \tilde{v}_\alpha$ to the gauge field kinetic energy. In the Einstein metric ${G^c}_{MN} = e^{-\Phi/2} G_{MN}$, the exact dilaton dependence of the kinetic energy of the gauge field $F_{\alpha MN}$, is [96]

$$L_{gauge} = -\frac{(2\pi)^3}{8\alpha'}\sqrt{G^c}\Sigma_\alpha \left(v_\alpha e^{-\Phi/2} + \tilde{v}_\alpha e^{\Phi/2} \right) \mathrm{tr} F_{\alpha MN} {F_\alpha}^{MN}. \tag{3.14}$$

So whenever one of the $\tilde{v}_\alpha$ is negative, there is a value of the dilaton for which the coupling constant of the corresponding gauge group diverges. This is believed to signal a phase transition associated with the appearance of tensionless strings [88, 89, 90]. This does not happen for the symmetric embedding discussed above since the perturbative gauge fields have $v_\alpha > 0$ and $\tilde{v}_\alpha = 0$ and the non-perturbative gauge fields have $v_\alpha = 0$ and $\tilde{v}_\alpha > 0$. Another kind of heterotic/heterotic duality may arise, however, in vacua where one may Higgs away that subset of gauge fields with negative $\tilde{v}_\alpha$, and be left with gauge fields with $v_\alpha = \tilde{v}_\alpha > 0$. This happens for the non-symmetric embedding $k = 14$ and the appearance of non-perturbative gauge fields is not required [80, 81, 82, 85, 86]. Despite appearances, it known from F-theory that the $k = 12$ and $k = 14$ models are actually equivalent [85, 86].

Vacua with $(N_+, N_-) = (2, 0)$ arising from Type IIB on $K3$ also have an M-theoretic description, in terms of compactification on T^5/Z_2 [87, 17].

4 Four dimensions

It is interesting to consider further toroidal compactification to four dimensions, replacing R^6 by $R^4 \times T^2$. Starting with a $K3$ vacuum in which the $E_8 \times E_8$ gauge symmetry is completely Higgsed, the toroidal compactification to four dimensions gives an $N = 2$ theory with the usual three vector multiplets S, T and U related to the four-dimensional heterotic string coupling constant and the area and shape of the T^2. When reduced to four dimensions, the six-dimensional string-string duality (3.10) becomes [8] an operation that exchanges S and T, so in the case of heterotic/heterotic duality we have a discrete $S - T$ interchange symmetry. This self-duality of heterotic string vacua does not rule out the the possibility that in $D = 4$ they are also dual to Type IIA strings compactified on Calabi-Yau manifolds. In fact, as discussed in [93], when the gauge group is completely Higgsed, obvious candidates are provided by Calabi-Yau manifolds with hodge numbers $h_{11} = 3$ and $h_{21} = 243$, since these have the same massless field content. Moreover, these manifolds do indeed exhibit the $S - T$ interchange symmetry [92, 91, 94]. Since the heterotic string on $T^2 \times K3$ also has R to $1/R$ symmetries that exchange T and U, one might expect a complete $S-T-U$ triality symmetry, as discussed in [66]. In all known models, however, the $T-U$ interchange symmetry is spoiled by non-perturbative effects [95, 98].

An interesting aspect of the Calabi-Yau manifolds X appearing in the duality between heterotic strings on $K3\times T^2$ and Type IIA strings on X, is that they can always be written in the form of a $K3$ fibration [92]. Once again, this ubiquity of $K3$ is presumably a consequence of the interpretation of the heterotic string as the $K3$ wrapping of a fivebrane. Consequently, if X admits *two* different $K3$ fibrations, this would provide an alternative explanation for heterotic dual pairs in four dimensions [83, 85, 86] and this is indeed the case for the Calabi-Yau manifolds discussed above.

5 Eleven to twelve: is it still too early?

The M-theoretic origin of the Type IIB string given in (xii) of section 2 seems to require going down to nine dimensions and then back up to ten. An obvious question, therefore, is whether Type IIB admits a more direct higher-dimensional explanation, like Type IIA. Already in 1987 it was suggested [99] that the $(1,1)$-signature worldsheet of the Type IIB string, moving in a $(9,1)$-signature spacetime, may be descended from a supersymmetric extended object with a $(2,2)$-signature worldvolume, moving in a $(10,2)$-signature spacetime. This idea becomes even more appealing if one imagines that the $SL(2,Z)$ of the Type IIB theory [13] might correspond to the modular group of a T^2 compactification from $D=12$ to $D=10$ just as the $SL(2,Z)$ of S-duality corresponds to the modular group of a T^2 compactification from $D=6$ to $D=4$ [8]. In view of our claims that $D=11$ is the maximum spacetime dimension admitting a consistent supersymmetric theory, however, this twelve dimensional idea requires some explanation. So let us begin by recalling the $D=11$ argument.

As a p-brane moves through spacetime, its trajectory is described by the functions $X^M(\xi)$ where X^M are the spacetime coordinates $(M=0,1,\ldots,D-1)$ and ξ^i are the worldvolume coordinates $(i=0,1,\ldots,d-1)$. It is often convenient to make the so-called "static gauge choice" by making the $D=d+(D-d)$ split

$$X^M(\xi)=(X^\mu(\xi),Y^m(\xi)), \tag{5.1}$$

where $\mu = 0, 1, \ldots, d-1$ and $m = d, \ldots, D-1$, and then setting

$$X^\mu(\xi) = \xi^\mu. \tag{5.2}$$

Thus the only physical worldvolume degrees of freedom are given by the $(D-d)$ $Y^m(\xi)$. So the number of on-shell bosonic degrees of freedom is

$$N_B = D - d. \tag{5.3}$$

To describe the super p-brane we augment the D bosonic coordinates $X^M(\xi)$ with anticommuting fermionic coordinates $\theta^\alpha(\xi)$. Depending on D, this spinor could be Dirac, Weyl, Majorana or Majorana-Weyl. The fermionic κ-symmetry means that half of the spinor degrees of freedom are redundant and may be eliminated by a physical gauge choice. The net result is that the theory exhibits a *d-dimensional worldvolume supersymmetry* [100] where the number of fermionic generators is exactly half of the generators in the original spacetime supersymmetry. This partial breaking of supersymmetry is a key idea. Let M be the number of real components of the minimal spinor and N the number of supersymmetries in D spacetime dimensions and let m and n be the corresponding quantities in d worldvolume dimensions. Since κ-symmetry always halves the number of fermionic degrees of freedom and (for $d > 2$) going on-shell halves it again, the number of on-shell fermionic degrees of freedom is

$$N_F = \frac{1}{2}\, mn = \frac{1}{4}\, MN. \tag{5.4}$$

Worldvolume supersymmetry demands $N_B = N_F$ and hence

$$D - d = \frac{1}{2}\, mn = \frac{1}{4}\, MN. \tag{5.5}$$

We note in particular that $D_{\max} = 11$ since $M = 32$ for $D = 11$ and we find the supermembrane with $d = 3$. For $D \geq 12$, $M \geq 64$ and hence (5.5) cannot be satisfied. Actually, the above argument is strictly valid only for p-branes whose worldvolume degrees of freedom are described by scalar supermultiplets. There are also p-branes with vector and/or antisymmetric tensor supermultiplets on the worldvolume [33, 34, 40], but repeating the argument still yields $D_{\max} = 11$ where we find a superfivebrane with $d = 6$ [14].

The upper bound of $D = 11$ is thus a consequence of the jump from $M = 32$ to $M = 64$ in going from $D = 11$ to $D = 12$. However, this jump can be avoided if one is willing to pay the price of changing the signature to $(10, 2)$ where it is possible to define a spinor which is both Majorana and Weyl. A naive application of the above bose-fermi matching argument then yields $D_{\max} = 12$ where we find an extended object with $d = 4$ but with $(2, 2)$ signature [99]. The chiral nature of this object then naturally suggests a connection with the Type IIB string in $D = 10$, although the T^2 compactification would have to be of an unusual kind in order to preserve the chirality. Moreover, the chiral $(N_+, N_-) = (1, 0)$ supersymmetry algebra in $(10, 2)$ involves the anti-commutator [101]

$$\{Q_\alpha, Q_\beta\} = \Gamma^{MN}{}_{\alpha\beta} P_{MN} + \Gamma^{MNPQRS}{}_{\alpha\beta} Z^{+}{}_{MNPQRS} \tag{5.6}$$

The absence of translations casts doubt on the naive application of the bose-fermi matching argument, and the appearance of the self-dual 6-form charge Z is suggestive of a sixbrane, rather than a threebrane.

Despite all the objections one might raise to a world with two time dimensions, and despite the above problems of interpretation, the idea of a $(2, 2)$ object moving in a $(10, 2)$ spacetime has recently been revived in the context of *F-theory* [75], which involves Type IIB compactification where the axion and dilaton from the RR sector are allowed to vary on the internal manifold. Given a manifold M that has the structure of a fiber bundle whose fiber is T^2 and whose base is some manifold B, then

$$F \ on \ M \equiv \ Type \ IIB \ on \ B \tag{5.7}$$

The utility of F-theory is beyond dispute and it has certainly enhanced our understanding of string dualities, but should the twelve-dimensions of F-theory be taken seriously? And if so, should F-theory be regarded as more fundamental than M-theory? Given that there seems to be no supersymmetric field theory with $SO(10, 2)$ Lorentz invariance [102], and given that the on-shell states carry only ten-dimensional momenta [75], the more conservative interpretation is that the twelfth dimension is merely a mathematical artifact and that F-theory should simply be interpreted as a clever way of compactifying the IIB string [103]. Time will tell.

6 Conclusion

The overriding problem in superunification in the coming years will be to take the Mystery out of M-theory, while keeping the Magic and the Membranes.

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