

THE WORLD IN ELEVEN DIMENSIONS:
A TRIBUTE TO OSKAR KLEIN¹

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Abstract

Current attempts to find a unified theory that would reconcile Einstein's General Relativity and Quantum Mechanics, and explain all known physical phenomena, invoke the Kaluza-Klein idea of extra spacetime dimensions. The best candidate is M-theory, which lives in eleven dimensions, the maximum allowed by supersymmetry of the elementary particles. We give a non-technical account.

An Appendix provides an updated version of Edwin A. Abbott's 1884 satire *Flatland: A Romance of Many Dimensions*. Entitled *Flatland, Modulo 8*, it describes the adventures of a superstring theorist, A. Square, who inhabits a ten-dimensional world and is initially reluctant to accept the existence of an eleventh dimension.

"Returning to my Ann Arbor attempts, I became immediately very eager to see how far the mentioned analogy reached, first trying to find out whether the Maxwell equations for the electromagnetic field together with Einstein's gravitational equations would fit into a formalism of five-dimensional Riemann geometry."

Oskar Klein, "From My Life in Physics"

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1 Physics of the new millennium

At the end of the last millennium, it became fashionable for certain pundits to declare the *End of Science*, on the grounds that all the most important scientific discoveries had already been made. To a physicist, this seems absurd because the two main pillars of twentieth century physics, namely Einstein's General Theory of Relativity and Quantum Mechanics, are mutually incompatible. At the microscopic level, general relativity fails to comply with the quantum rules which govern the behavior of the elementary particles; while on the macroscopic scale, the black holes of Einstein's theory are threatening the very foundations of quantum mechanics. Something big has to give. This augurs less the bleak future of diminishing returns predicted by the millennial Jeremiahs and more another scientific revolution.

Many physicists believe that this revolution is already under way with the theory of *Superstrings*. As their name suggests, superstrings are one-dimensional string-like objects [1]. Just like violin strings, these relativistic strings can vibrate and each mode of vibration corresponds to a different elementary particle. One strange feature of superstrings is that they live in a universe with nine space dimensions and one time dimension. Since the world around us seems to have only three space dimensions, the extra six would have to be curled up to an unobservably small size (or else rendered invisible in some other way) if the theory is to be at all realistic. This idea of extra dimensions will be an important theme in this lecture. Many of you in this audience will be familiar with superstring theory following the millennial edition of the annual international superstrings conference "Strings 2000" hosted last year here at the University of Michigan [2]. See Figure 1.

What may be less familiar to the general public is that superstring theory has recently been superseded by a deeper and more profound new theory, called "M-theory" [3]. M-theory involves membrane-like extended objects with two space dimensions and five space dimensions that themselves live in a universe with eleven spacetime dimensions [4] (ten space and one time). As we shall see, it not only subsumes all of the new ideas of superstring theory but also revives older ideas on eleven-dimensional supergravity. See Figure 2. New evidence in favor of this theory is appearing daily on the internet and represents the most exciting development^a in the subject since 1984 when the superstring revolution first burst on the scene.

2 The fundamental constituents of matter

Theoretical physicists like to ask the big questions: How did the Universe begin? What are its fundamental constituents? What are the laws of nature that govern these constituents?

The smallest constituents of matter are, by definition, the *elementary particles*. But what is an elementary particle, exactly? How do we know when we have

^aIndeed, several of the stores in Ann Arbor changed their name to "M-dens" and many students at the University of Michigan can be seen wearing M-theory tee shirts, M-theory baseball caps and drinking from M-theory mugs.

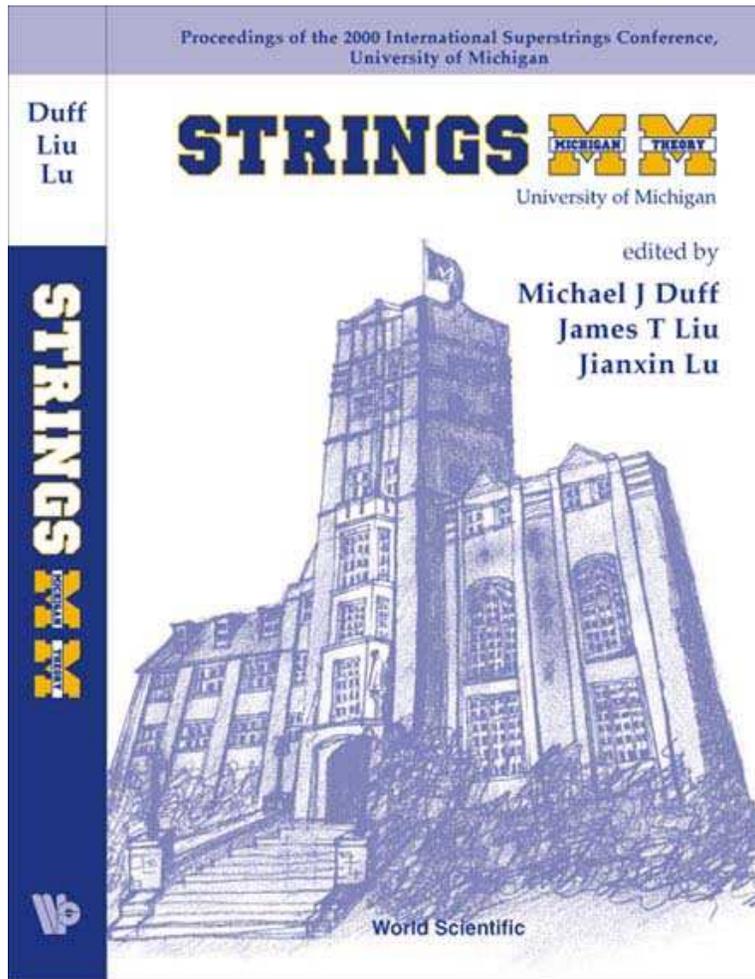


Figure 1: The Strings 2000 Conference.

reached the bottom line? Well, it turns out to be easier to say what an elementary particle is not. For example, as illustrated in Figure 3, a human being (1 meter) is not elementary because he or she is made out of molecules. A molecule (10^{-8} meters) is not elementary because it is made out of atoms. An atom (10^{-10} meters) is not elementary because it is made out of electrons orbiting a nucleus which is made out of protons and neutrons (collectively called *nucleons*). Nucleons (10^{-15} meters) are not elementary because they are made out of smaller constituents called *quarks*. Finally, however, physicists believe that these quarks, together with other particles called *leptons*, of which the electron is an example, are indeed truly elementary. They appear to have no size of their own and, as far as we can tell (which is down to about 10^{-18} meters), behave like geometrical points.

Because of this, elementary particle physics is qualitatively different from all other branches of science. In his or her everyday research, the biologist can take for granted all of chemistry; the chemist can take for granted all of atomic physics;

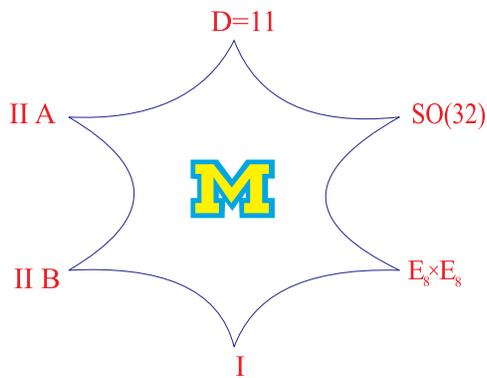


Figure 2: M-theory subsumes eleven-dimensional supergravity and the five ten-dimensional superstring theories.

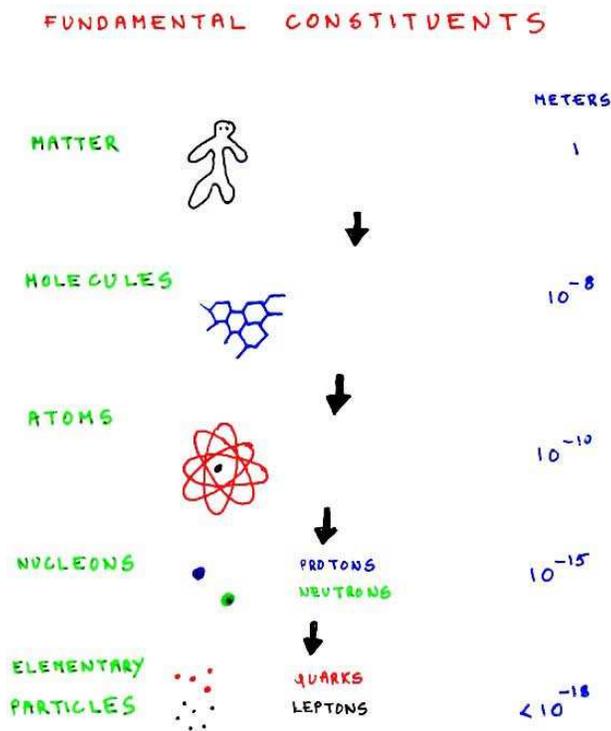


Figure 3: Fundamental constituents.

<i>I</i>	ν_e	e^-	u	d
<i>II</i>	ν_μ	μ^-	c	s
<i>III</i>	ν_τ	τ^-	t	b
	$Q = 0$	$Q = -1$	$Q = 2/3$	$Q = -1/3$

Table 1: Three families of quarks and leptons, with each quark coming in three colors (red, green and blue).

the atomic physicist can take for granted all of nuclear physics and the nuclear physicist can take for granted all of elementary particle physics. Particle physicist are different, however, because they can take nothing for granted. There is no higher court of appeal. Someone once said that if you compare scientific research to a game of chess, then most scientists are trying to become masters of the game. But the particle physicist is still trying to figure out what the rules are!

Let us now take a preliminary look at the matter particles as displayed in Table 1. Theoretical consistency, borne out by experiment, requires that quarks and leptons come in families. For reasons we do not yet understand theoretically, the number of families is exactly three. Ordinary matter that we encounter on earth outside of particle accelerators is composed only of the first family, which consists of an electron neutrino with electric charge $Q = 0$, an electron with $Q = -1$, an *up* quark with $Q = +2/3$ and a *down* quark with $Q = -1/3$. These particles are denoted ν_e , e , u and d respectively. All members have a non-zero rest mass except the neutrino which therefore travels at the speed of light. Apart from being heavier, the second and third families are identical replicas of the first. The second consists of a muon neutrino ν_μ , a muon μ , a *charm* quark c and a *strange* quark s . The third consists of a tau neutrino ν_τ , a tau τ , a *top* quark t and a *bottom* quark b . The six different types of quark are known as the six *flavors*. However, each quark also comes in three different *colors* labeled red, green and blue after the three primary colors. This is of course just an analogy, but it is a good one because just we can form *colorless* combinations from red, green and blue, so we can combine colored quarks to form colorless bound states. In fact, it seems to be a fact of nature that the only particles we actually observe are such colorless combinations. In particular, we never observe free quarks. It is a prediction of quantum field theory that for every particle there is an antiparticle with the same mass but opposite charge. So in addition to all the above particles, we also have their antiparticles. These are denoted by a bar, so that an anti-up quark, for example would be \bar{u} . Particles that were previously believed to be elementary are now known to be built out of these basic entities. For example, the proton ($Q = 1$) is a uud combination, the neutron ($Q = 0$) is a ddu and the positively charged pion ($Q = 1$) is a $u\bar{d}$.

These fundamental building blocks of nature are held together by four fundamental forces, listed in Table 2 in order of diminishing strength. The strong nuclear force holds the protons and neutrons together inside the atomic nucleus; the electromagnetic force is responsible for all electric and magnetic phenomena and for light (light is an electromagnetic wave): the weak nuclear force is responsible for

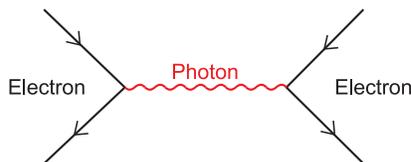


Figure 4: Feynman diagram representing the force between two electrons.

radioactivity, and lastly we have the gravitational force which is most familiar in our everyday lives. As we see, the gravitational force is incredibly weak compared to the other three forces and, for this reason, most text-books will tell you that gravity is of no consequence at the subatomic scale. However, I will try to convince you that most text-books are wrong: not only will gravity prove to be important but all the theoretical evidence suggests that it is actually at the root of everything else.

What exactly do we mean by a *force*? Over the years, our answer to this question has become more and more sophisticated. Nowadays, we employ quantum field theory, which is what we are led to when we try and combine quantum mechanics with special relativity. According to quantum field theory, each force is associated with a *force carrier*. For example, when an electron repels another electron, it does so by exchanging the fundamental quantum of electromagnetism, the *photon*, as in Figure 4. By the way, this picture is an example of what are called *Feynman diagrams*, named after Richard Feynman, who invented them. The study of the interactions between electrons and photons is known as *quantum electrodynamics* or *QED* and, in terms of the accuracy with which it agrees with experiment, it is the most successful theory ever invented. For example, a measure of the electron's magnetism is the g-factor. According to QED, it is given by

$$g/2 = 1.001159652190 \tag{1}$$

whereas the experimental value is

$$g/2 = 1.001159652193 \tag{2}$$

Because of this success, physicists have invented other force carriers to account for the other forces. The carriers of the strong force are known as *gluons* and they act on quarks. The study of the interactions between colored quarks and gluons is known as *quantum chromodynamics* or *QCD*. The existence of the gluon was established indirectly in 1978. The carriers of the weak force are known as W and Z bosons and, together with the photon, they act on both quarks and leptons. For example, the phenomenon of radioactivity arise from the so-called β -decay of a neutron into a proton, an electron and an antineutrino as shown by the Feynman diagram in Figure 5. This interaction is mediated by the exchange of a W boson. The W boson was discovered at CERN in 1982 and the Z boson, the following year. The collective study of the interactions of quarks and leptons with photons, W and Z bosons is known as *electro-weak theory*.

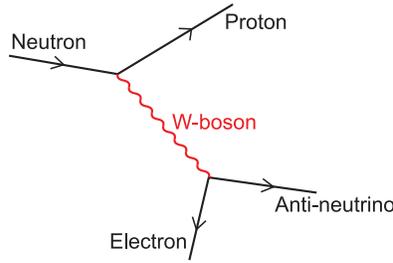


Figure 5: Feynman diagram representing the radioactive decay of a neutron.

Finally, physicists have coined the word *graviton* to describe the force carrier of gravity, which acts on every other matter particle and force carrier. This is, at the moment, only a hypothetical particle and is unlikely to be detected experimentally for many years to come.

So far, we have not discussed the origin of mass within the Standard Model. How come the neutrino is massless but the electron is not? How come the photon is massless but the W is not? It turns out that it is necessary to introduce a third kind of particle, called the *Higgs boson* after the Scottish theorist Peter Higgs, whose job it is cleverly to give masses to the particles that need masses (like the e and the W) while not giving masses to those that do not (like the ν and the γ). Thus the Higgs is neither a force carrier nor a matter particle.

Each elementary particle carries an intrinsic angular momentum or *spin*, s , which can either be an integer ($s = 0, 1, 2, \dots$), in which case it is called a *boson*, or an odd half-integer ($s = 1/2, 3/2, 5/2, \dots$), in which case it is called a *fermion*. The force carriers (gluons, photons, W and Z) are all bosons with $s = 1$; the matter particles (quarks and leptons) are all fermions with $s = 1/2$; the Higgs is an $s = 0$ boson. Bosons and fermions behave very differently. For example, fermions obey the *exclusion principle* of Wolfgang Pauli, which states that no two fermions can occupy the same quantum state, whereas bosons do not. They are said to obey *opposite statistics*.

The *Standard Model* is the name we give to the collection of strong, weak and electromagnetic interactions. It describes the interaction of the matter particles, force carriers and Higgs. It is consistent with every experiment that has yet been performed^b, but it also predicts other effects which have not yet been seen. In particular, there is still no direct experimental evidence for the Higgs boson. An important goal of future experiments at Fermilab in Chicago and the Large Hadron Collider at CERN in Geneva, will be to hunt for the Higgs.

Three of the heroes of the Standard Model, Sheldon Glashow (who won the 1979 Nobel Prize for explaining how to unify the weak and electromagnetic forces), Martinus Veltman (who won the 1999 Nobel prize for showing how the theory is physically and mathematically consistent) and Peter Higgs (without whose boson everything in the universe would be massless, and who therefore bears a heavy

^bRecent experiments have indicated that the neutrino has a very small but non-zero mass, but this would require only relatively mild modifications to the Standard Model.

responsibility, in more ways than one) will be speaking at “2001: A Spacetime Odyssey”, the Inaugural Conference of the Michigan Center for Theoretical Physics to be held here in Ann Arbor in May [5]. See Figure 6.

<i>FORCE</i>	<i>STRENGTH</i>	<i>CARRIER</i>	<i>ACTS ON</i>
<i>strong</i>	1	<i>gluon</i>	<i>quarks</i>
<i>electromagnetic</i>	10^{-2}	<i>photon</i>	<i>quarks and leptons</i>
<i>weak</i>	10^{-5}	<i>W and Z bosons</i>	<i>quarks and leptons</i>
<i>gravity</i>	10^{-38}	<i>graviton</i>	<i>everything</i>

Table 2: The fundamental forces.

3 What about gravity?

Why is it so difficult to incorporate the fourth force of gravity into quantum physics? Our modern understanding of the gravitational force dates back to Einstein’s 1916 theory of general relativity which asserts that the laws of physics should be the same to *all* observers, not merely those in uniform relative motion as in special relativity. According to this picture, gravity is not a *force* at all. All bodies are following straight line trajectories, but in a *curved spacetime*. (Note that it is not merely three-dimensional space which is curved, but the four-dimensional spacetime continuum.) A classic illustration of this idea is provided by the bending of light by the Sun, as shown in Figure 7. At first sight, these classical geometrical ideas seem to be at odds with the quantum picture of force-carrying gravitons. In recent times, however, we have come to appreciate that these two pictures are complementary. Indeed, the emphasis now is to explain the other three forces in geometrical terms as well.

The goal of modern theoretical physics is to find an all-embracing “Theory of Everything” that would unite all four forces. But if current thinking about M-theory is correct, this will require three radical ideas:

- 1) Supersymmetry
- 2) Extra spacetime dimensions
- 3) Extended objects

4 Supersymmetry

Central to the understanding of modern theories of the fundamental forces is the idea of *symmetry*: under certain changes in the way we describe the basic quantities, the laws of physics are nevertheless seen to remain unchanged. For example, the result of an experiment should be the same whether we perform it today or tomorrow; this symmetry is called *time translation invariance*. It should also be the same before and after rotating our experimental apparatus; this symmetry is

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Image: Gessau Pillars in M16, Eagle Nebula, Hubble Space Telescope, PRC91-644, STScI OPO, November 2, 1995. J. Hester and B. Scowen (Arizona State University), NASA

Figure 6: 2001: A Spacetime Odyssey.

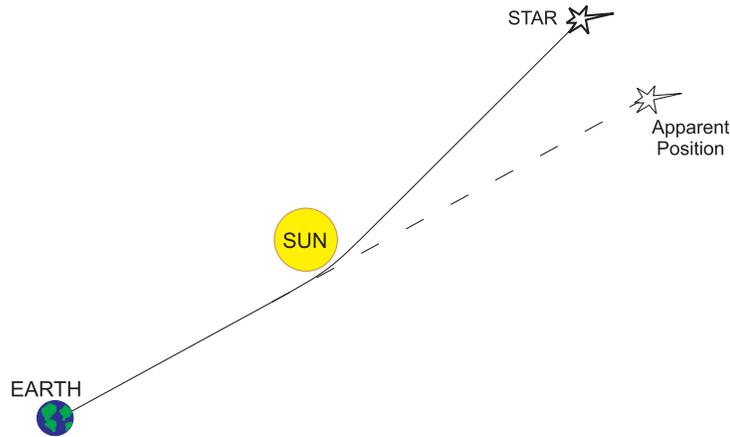


Figure 7: The bending of light by the Sun.

called *rotational invariance*. Both of these are examples of *spacetime symmetries*. Indeed, Einstein’s general theory of relativity is based on the requirement that the laws of physics should be invariant under *any* change in the way we describe the positions of events in spacetime. In the Standard Model of the strong, weak and electromagnetic forces there are other kinds of *internal* symmetries that allow us to change the roles played by different elementary particles such as electrons and neutrinos, for example. In Grand Unified Theories, which have not yet received the same empirical support as the Standard Model, the laws remain unchanged even when we exchange the roles of the *quarks* and electrons. Thus it is that the greater the unification, the greater the symmetry required. The Standard Model symmetry replaces the three fundamental forces: strong, weak and electromagnetic, with just two: the strong and electroweak. Grand unified symmetries replace these two with just one strong-electroweak force. In fact, it is not much of an exaggeration to say that the search for the ultimate unified theory is really a search for the right symmetry.

At this stage, however, one might protest that some of these internal symmetries fly in the face of experience. After all, the electron is very different from a neutrino: the electron has a non-zero mass whereas the neutrino is massless. Similarly, the electrons which orbit the atomic nucleus are very different from the quarks out of which the protons and neutrons of the nucleus are built. Quarks feel the strong nuclear force which holds the nucleus together, whereas electrons do not. These feelings are, in a certain sense, justified: the world we live in does not exhibit the symmetries of the Standard Model nor those of Grand Unified Theories. They are what physicists call “broken symmetries”. The idea is that these theories may exist in several different *phases*, just as water can exist in solid, liquid and gaseous phases. In some of these phases the symmetries are broken but in other phases, they are exact. The world we inhabit today happens to correspond to the broken-symmetric phase, but in conditions of extremely high energies or extremely high temperatures, these symmetries may be restored to their pristine form. The early

stages of our universe, shortly after the Big Bang, provide just such an environment. Looking back further into the history of the universe, therefore, is also a search for greater and greater symmetry. The ultimate symmetry we are looking for may well be the symmetry with which the Universe began.

M -theory, like string theory before it, relies crucially on the idea, first put forward in the early 1970s, of a spacetime *supersymmetry* which exchanges bosons and fermions. Unbroken supersymmetry would require that every elementary particle we know of would have an unknown super-partner with the same mass but obeying the opposite statistics: for each boson there is a fermion; for each fermion a boson. Spin 1/2 quarks partner spin 0 *squarks*, spin 1 photons partner spin 1/2 *photinos*, and so on. In the world we inhabit, of course, there are no such equal mass partners and bosons and fermions seem very different. Supersymmetry, if it exists at all, is clearly a broken symmetry and the new supersymmetric particles are so heavy that they have so far escaped detection. At sufficiently high energies, however, supersymmetry may be restored. Another challenge currently facing high-energy experimentalists at Fermilab and CERN is the search for these new supersymmetric particles. The discovery of supersymmetry would be one of the greatest experimental achievements and would completely revolutionize the way we view the physical world^c.

Symmetries are said to be *global* if the changes are the same throughout spacetime, and *local* if they differ from one point to another. The consequences of *local* supersymmetry are even more far-reaching: it predicts gravity. Thus if Einstein had not already discovered General Relativity, local supersymmetry would have forced us to invent it. In fact, we are forced to a *supergravity* in which the graviton, a spin 2 boson that mediates the gravitational interactions, is partnered with a spin 3/2 *gravitino*. This is a theorist's dream because it confronts the problem from which both general relativity and grand unified theories shy away: neither takes the other's symmetries into account. Consequently, neither is able to achieve the ultimate unification and roll all four forces into one. But local supersymmetry offers just such a possibility, and it is this feature above all others which has fuelled the theorist's belief in supersymmetry in spite of thirty years without experimental support.

Supergravity has an even more bizarre feature, however, it places an upper limit of *eleven* on the dimension of spacetime! There are two different ways to see this. The original explanation relies on the belief that there are no consistent ways to describe massless particles with spins greater than two, the spin of the graviton. Yet the mathematics of supersymmetry says that spins greater than two (and more than one graviton) would have to appear in dimensions greater than eleven, thus leading to a contradiction. The second explanation, to which we shall return in section 10, does not rely on the absence of higher spins, but says the maximum dimension in which there exists supersymmetric objects such as particles, strings

^cThis will present an interesting dilemma for those pundits who are predicting the *End of Science* on the grounds that all the important discoveries have already been made. Presumably, they will say "I told you so" if supersymmetry is not discovered, and "See, there's one thing less left to discover" if it is.

and membranes is eleven, where in fact we find a supermembrane.

We are used to the idea that space has three dimensions: height, length and breadth; with time providing the fourth dimension of *spacetime*. Indeed this is the picture that Einstein had in mind in 1916 when he proposed general relativity. But in the early 1920's, in their attempts to unify Einstein's gravity and Maxwell's electromagnetism, Theodor Kaluza and Oskar Klein suggested that spacetime may have a hidden fifth dimension. This Kaluza-Klein theory, and its higher dimensional generalizations, are thus tailor-made for supersymmetry.

5 The fifth dimension

In 1864 James Clerk Maxwell introduced the equations that describe the electromagnetic field. Albert Einstein later realized in 1905 that Maxwell's equations obey the principle of special relativity, that the laws of physics should be the same to all observers who are in uniform relative motion. In special relativity, which treats time as a fourth dimension, time t and the space coordinates x, y, z are collectively denoted x^μ where the index μ runs over 0, 1, 2, 3. The 0 refers to time and 1, 2, 3 to the three space coordinates (x, y, z) :

$$(x^0, x^1, x^2, x^3) = (t, x, y, z) \quad (3)$$

In this modern notation, Maxwell's electromagnetic field also has four components collectively denoted $A_\mu(x)$. The field depends on its position in spacetime and so is a function of the 4 spacetime coordinates x^μ .

In 1916, Einstein introduced the principle of general relativity, that the laws of physics should be the same to *all* observers. This necessitates a gravitational field with two indices, $g_{\mu\nu}(x)$, which also has the geometrical interpretation of a metric tensor, the quantity that describes the infinitesimal distance ds between two points in four-dimensional spacetime.

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu \quad (4)$$

Note that the Euclidean geometry of flat spacetime must be replaced by the Riemannian geometry of curved spacetime for which the metric tensor is itself a function of the spacetime coordinates.

In 1919, therefore, in the search for a unified theory, it was natural to attempt to combine Maxwell's electromagnetism with Einstein's gravity, since the other two nuclear forces were not as well understood. This the German-Polish mathematician Theodor Kaluza was able to do by the ingenious device of postulating a fifth dimension with coordinate θ . The five coordinates are denoted collectively x^M where the index M runs over 0, 1, 2, 3, 4.

$$(x^0, x^1, x^2, x^3, x^4) = (t, x, y, z, \theta) \quad (5)$$

He imagined a five dimensional Riemannian geometry with metric tensor $\hat{g}_{MN}(x)$ which describes the infinitesimal distance $d\hat{s}$ between two points in this five-dimensional

spacetime

$$d\hat{s}^2 = \hat{g}_{MN}(x)dx^M dx^N \quad (6)$$

He then made a 4 + 1 split

$$\hat{g}_{MN} = \begin{pmatrix} g_{\mu\nu} + \Phi A_\mu A_\nu & \Phi A_\mu \\ \Phi A_\nu & \Phi \end{pmatrix} \quad (7)$$

and identified $g_{\mu\nu}(x)$ with Einstein's gravitational field and $A_\mu(x)$ with Maxwell's electromagnetic field. This was all before the advent of quantum field theory, but nowadays we would refer to $g_{\mu\nu}$ as the spin 2 graviton, A_μ as the spin 1 photon and Φ as the spin 0 dilaton^d.

Of course, it is not enough to call A_μ by the name photon, we must demonstrate that it obeys the right equations and here we see the Kaluza miracle at work. If we substitute (7) into the five dimensional field equations, not only do we recover the correct Einstein equations for $g_{\mu\nu}$, but also the Maxwell equations for A_μ . So Maxwell's theory of electromagnetism was seen to be a consequence of Einstein's general relativity, provided you are willing to buy the idea of an extra spacetime dimension.

By the way, Kaluza's son, who still teaches mathematics in Germany, recalls that his father belonged to that school of theoreticians who believed that everything in nature could be derived from pure thought without the need for experiment. Consequently, he learned to swim from a textbook. He would lie on the living room couch with the book in one hand and practice his strokes with the other. He then proceeded to walk to the nearest lake and swim across it. Personally, I find it easier to believe in a fifth dimension than in the veracity of that particular story.

Attractive though Kaluza's idea was, it suffered from two obvious drawbacks. First, although the indices were allowed to range over 0, 1, 2, 3, 4, for no very good reason the dependence of the fields on the extra coordinate θ was suppressed. Secondly, if there is a fifth dimension why haven't we seen it? The resolution of both these problems was supplied by Oskar Klein^e in 1926 [6, 7]. Klein insisted on treating the extra dimension seriously but assumed the fifth dimension to have circular topology so that the coordinate θ is periodic, $0 \leq \theta \leq 2\pi$. It is difficult to envisage a spacetime with this topology but a simpler two-dimensional analogy is provided by a garden hose: at large distances it looks like a line but closer inspection reveals that at every point on the line there is a little circle. So it was that Klein suggested that there is a little circle at each point in four-dimensional spacetime. See Figure 8.

^dThis field was considered an embarrassment in 1919, and was (inconsistently) set equal to zero. However, it was later revived and subsequently stimulated Brans-Dicke theories of gravity. The dilaton also plays a crucial role in superstring and M-theory.

^eNot to be confused (as the *Oxford Dictionary of Physics* is) with Felix Klein (no relation), inventor of the Klein bottle.

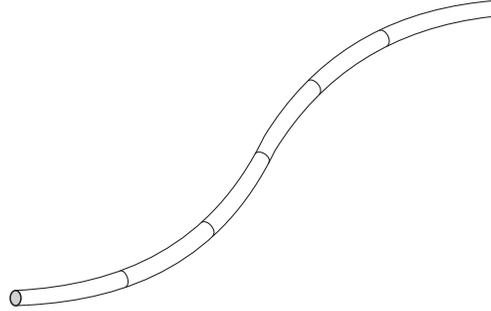


Figure 8: A garden hose. At large distances, it looks like a line; closer inspection reveals that at each point of the line there is a little circle.

The periodicity in θ means that the fields $\hat{g}_{MN}(x, \theta)$ may be expanded in the form

$$\hat{g}_{MN}(x, \theta) = \sum_{n=-\infty}^{n=\infty} \hat{g}_{MN(n)}(x) \exp(in\theta), \quad (8)$$

The $n = 0$ modes in (8) are just Kaluza's graviton, photon and dilaton. If we now include the θ -dependence via the $n \neq 0$ modes, however, we find an infinite tower of charged, massive spin 2 particles with charges e_n given by

$$e_n = ne, \quad (9)$$

and masses m_n given by

$$m_n = |n|/R \quad (10)$$

where R is the radius of the circle and

$$e^2 = 16\pi G/R^2 \quad (11)$$

where G is Newton's constant of gravitation. Thus Klein explained (for the first time) the empirical fact that all observed particles come with an electric charge which is an integer multiple of a fundamental charge e , in other words, why electric charge is *quantized*. Of course, if we identify this fundamental unit of charge with the charge on the electron, then we are forced to take the radius of the circle to be very small: the Planck size 10^{-35} meters; much smaller than the 10^{-18} meters achievable by current particle accelerators. This satisfactorily accords with our everyday experience of living in four spacetime dimensions.

6 Oskar Klein

As the quote on the cover page shows, the idea for a fifth dimension came to Klein during his stay as an assistant professor at the University of Michigan, 1923-25. He was hired by H. M. Randall on the recommendation of the father of quantum theory, Niels Bohr, with whom he had been working in Copenhagen. He was a



Figure 9: Left to right: Oskar Klein, George Uhlenbeck and Samuel Goudsmit (American Institute of Physics).

contemporary of two other famous Michigan faculty members, George Uhlenbeck and Samuel Goudsmit, discoverers of electron spin^f. See Figure 9.

Klein was also the thesis advisor of another distinguished Michigan faculty member, David Dennison, discoverer of proton spin. As Dennison recalls [8], Klein proved to be hard task master. See Figure 10.

Klein was not at first aware of Kaluza's earlier idea but learned about it from Wolfgang Pauli to whom he had shown his manuscript. Ever the gentleman, he generously acknowledges Kaluza's prior claim, but as we have noted above, it was Klein who really took seriously the extra dimension with all its implications, and by assigning it the topology of a circle, provided the first explanation of electric charge quantization.

Klein was born in Sweden in 1894 and in 1994 the Nobel Committee of the Royal Swedish Academy of Sciences organized The Oskar Klein Centenary Symposium in Stockholm, at which I was privileged to deliver a review of Kaluza-Klein theory [9]. See Figure 11. So I feel doubly privileged to be delivering this Oskar Klein Professorship Inaugural Lecture here today.

Klein was responsible for many other important discoveries, including the Klein paradox, the Klein-Gordon equation and the Klein-Nishina formula. A discussion of these and more on his life and times may be found in the recollections of that able historian of physics Abraham Pais [10, 11] and of my friend and colleague Stanley Deser [12] who is married to Klein's daughter Elsbeth.

^fVisitors to the Michigan Center for Theoretical Physics (<http://www.umich.edu/~mctp/>) are invited to view a collection of memorabilia and photographs of Klein (with Bohr, Goudsmit, Uhlenbeck and others) in the Oskar Klein Conference Room, 3481 Randall Laboratory.



DENNISON: “Professor Klein, you would write it one way but I would write it a different way.”



KLEIN: “No, there is only one way to write a scientific paper. Each sentence must be rigorous and correct and the thought must follow smoothly from one sentence to the next!”

Figure 10: Dennison’s recollection of his PhD thesis.

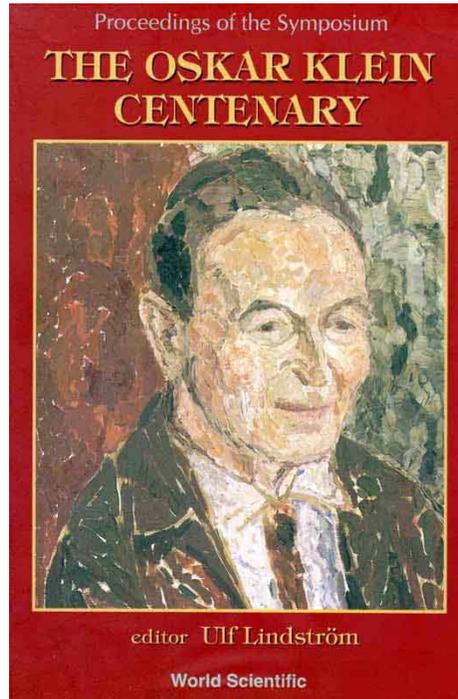


Figure 11: The Oskar Klein Centenary, 1994

7 Eleven dimensional supergravity

The Kaluza-Klein idea was forgotten for many years but was revived in the early 1980s when it was realized by Eugene Cremmer, Bernard Julia and Joel Scherk from the Ecole Normale in Paris that supergravity not only permits up to seven extra dimensions, but in fact takes its simplest and most elegant form when written in its full eleven-dimensional glory. Moreover, the kind of four-dimensional picture we end up with depends on how we curl up or *compactify* these extra dimensions: maybe seven of them would allow us to derive, a la Kaluza-Klein, the strong and weak forces as well as the electromagnetic [14]. The four dimensional theory we end up after such a compactification describes a spin 2 graviton and spin 3/2 gravitinos interacting with a collection of spin 1, spin 1/2 and spin 0 particles. This collection will depend on the choice of the compact seven-dimensional space. So the question was whether there was a choice that yielded the gluons, W and Z-bosons, photons, quarks, leptons and Higgs of the Standard Model. In the end, however, eleven dimensional supergravity fell out of favor for a couple of reasons.

First, an important feature of the real world which is incorporated into both the Standard Model and Grand Unified Theories is that Nature is *chiral*: the weak nuclear force distinguishes between right and left. However, as emphasized by Edward Witten of the Institute for Advanced Study in Princeton, among others, it is impossible via conventional Kaluza-Klein techniques to generate a chiral theory from a non-chiral one and unfortunately, eleven-dimensional supergravity, in

common with any *odd*-dimensional theory, is itself non-chiral.

Secondly, despite its extra dimensions and despite its supersymmetry, eleven-dimensional supergravity is still a *quantum field theory* and runs into the problem from which all such theories suffer: the quantum mechanical probability for certain processes yields the answer *infinity*, signalling a breakdown of the theory. There is a natural energy scale associated with any quantum theory of gravity. Such a theory combines three ingredients each with their own fundamental constants: Planck's constant h (quantum mechanics), the velocity of light c (special relativity) and Newton's gravitational constant G (gravity). From these we can form the so-called Planck mass $m_P = \sqrt{\hbar c/G}$, equal to about 10^{-8} kilograms, and the Planck energy $m_P c^2$, equal to about 10^{19} GeV. (GeV is short for giga-electron-volts= 10^9 electron-volts, and an electron-volt is the energy required to accelerate an electron through a potential difference of one volt.) From this we conclude that the energy at which Einstein's theory, and hence eleven-dimensional supergravity, breaks down is the Planck energy. On the scale of elementary particle physics, this energy is enormous⁸ : the world's most powerful particle accelerators can currently reach energies of only 10^4 GeV. So it seemed in the early 1980s that we were looking for a fundamental theory which reduces to Einstein's gravity at low energies, which describes Planck mass particles and which is supersymmetric. Whatever it is, it cannot be a quantum field theory because we already know all the supersymmetric ones and they do not fit the bill.

8 Ten-dimensional superstrings

For both these reasons, attention turned to ten-dimensional superstring theory. The idea that the fundamental stuff of the universe might not be pointlike elementary particles, but rather one-dimensional strings had been around from the early 1970s. Just like violin strings, these relativistic strings can vibrate and each elementary particle: graviton, gluon, quark and so on, is identified with a different mode of vibration. However, this means that there are *infinitely many* elementary particles. Fortunately, this does not contradict experiment because most of them, corresponding to the higher modes of vibration, will have masses of the order of the Planck mass and above and will be unobservable in the direct sense that we observe the lighter ones. Indeed, an infinite tower of Planck mass states is just what the doctor ordered for curing the non-renormalizability disease. In fact, because strings are *extended*, rather than pointlike, objects, the quantum mechanical probabilities involved in string processes are actually *finite*. Moreover, when we take the *low-energy limit* by eliminating these massive particles through their equations of motion, we recover a ten-dimensional version of supergravity which incorporates Einstein's gravity. Now ten-dimensional theories, as opposed to eleven-dimensional ones, also admit the possibility of *chirality*. The reason that everyone had still

⁸For this reason, incidentally, the *End of Science* brigade like to claim that, even if we find the right theory of quantum gravity, we will never be able to test it experimentally! As I will argue shortly, however, this view is erroneous.

not abandoned eleven-dimensional supergravity in favor of string theory, however, was that the realistic-looking Type *I* string, which seemed capable of incorporating the Standard Model of particle physics, seemed to suffer from inconsistencies or *anomalies*, whereas the consistent non-chiral Type *IIA* and chiral Type *IIB* strings did not seem realistic.

Then came the September 1984 superstring revolution. First, Michael Green from Queen Mary College, London, and John Schwarz from the California Institute of Technology showed that the Type *I* string was free of anomalies provide the group was uniquely $SO(32)$ where $O(n)$ stands for *orthogonal* $n \times n$ matrices. They suggested that a string theory based on the exceptional group $E_8 \times E_8$ would also have this property. Next, David Gross, Jeffrey Harvey, Emil Martinec and Ryan Rohm from Princeton University discovered a new kind of heterotic (hybrid) string theory based on just these two groups: the $E_8 \times E_8$ heterotic string and the $SO(32)$ heterotic string, thus bringing to *five* the number of consistent string theories. Thirdly, Philip Candelas from the University of Texas, Austin, Gary Horowitz and Andrew Strominger from the University of California, Santa Barbara and Witten showed that these heterotic string theories admitted a Kaluza-Klein compactification from ten dimensions down to four. The six-dimensional compact spaces belonged to a class of spaces known to the pure mathematicians as *Calabi-Yau manifolds*. The resulting four-dimensional theories resembled quasi-realistic Grand Unified Theories with chiral representations for the quarks and leptons! So at last we had a consistent quantum theory of gravity that might even explain the Standard Model of particle physics! Everyone dropped eleven-dimensional supergravity like a hot brick. The mood of the times was encapsulated by Nobel Laureate Murray Gell-Mann (inventor of the quarks) in his closing address at the 1984 Santa Fe Meeting, when he said: “Eleven Dimensional Supergravity (Ugh!)”.

9 The liberty of doubt

So ten dimensions were riding high but eleven dimensions were in the doghouse. Nevertheless, a small band of theorists clung to the idea that eleven dimensions must somehow feature in the final theory. Notwithstanding the euphoria that surrounded string theory, they allowed themselves the *liberty of doubt* about string theory as it was currently formulated and began to ask some awkward questions [15]:

* *The uniqueness problem*

Theorists love *uniqueness*; they like to think that the ultimate *Theory of Everything* [16] will one day be singled out, not merely because all rival theories are in disagreement with experiment, but because they are mathematically inconsistent. In other words, that the universe is the way it is because it is the only possible universe. But string theories are far from unique. Already in ten dimensions there are five mathematically consistent theories as shown in Figure 2: the Type *I* $SO(32)$, the heterotic $SO(32)$, the heterotic $E_8 \times E_8$, the Type *IIA* and the Type *IIB*. (Type *I* is an *open* string in that its ends are allowed to move freely in spacetime;



A study of the history of science-not the history of philosophy-shows that the natural attitude of a scientist is to be inspired by the great predecessors, just as they themselves were by their predecessors, but always taking the liberty of doubt when there are reasons for doubt

Oskar Klein, "From My Life in Physics"

Figure 12: The liberty of doubt

the remaining four are *closed strings* which form a closed loop.) If we are looking for a unique all-embracing theory, this seems like an embarrassment of riches.

The situation becomes even worse when we consider compactifying the extra six dimensions. There seem to be billions of different ways of compactifying the string from ten dimensions to four (billions of different Calabi-Yau manifolds) and hence billions of competing predictions of the real world (which is like having no predictions at all). This aspect of the uniqueness problem is called the *vacuum-degeneracy problem*. One can associate with each different phase of a physical system a *vacuum state*, so called because it is the quantum state corresponding to no real elementary particles at all. However, according to quantum field theory, this vacuum is actually buzzing with virtual particle-antiparticle pairs that are continually being created and destroyed and consequently such vacuum states carry energy. The more energetic vacua, however, should be unstable and eventually decay into a (possibly unique) stable vacuum with the least energy, and this should describe the world in which we live. Unfortunately, all these Calabi-Yau vacua have the same energy and the string seems to have no way of preferring one to the other. By focusing on the fact that strings are formulated in ten spacetime dimensions and that they unify the forces at the Planck scale, many critics of string theory fail to grasp this essential point. The problem is not so much that strings are unable to produce four-dimensional models like the Standard Model with quarks and leptons held together by gluons, W -bosons, Z bosons and photons and of the kind that can be tested experimentally in current or foreseeable accelerators. On the contrary, string theorists can dream up literally billions of them! The problem is that they have no way of discriminating between them. What is lacking is some dynamical mechanism that would explain why the theory singles out one particular Calabi-Yau manifold and hence why we live in one particular vacuum; in other words, why the world is the way it is. Either this problem will not be solved, in which case string theory will fall by the wayside like a hundred other failed theories, or else it will be solved and string theory will be put to the test experimentally. Neither string theory nor M -theory is relying for its credibility on building thousand-light-year accelerators capable of reaching the Planck energy, as some *End-of-Science* Jeremiahs have suggested.

* *The dimension problem*

An apparently different reason for having mixed feelings about superstrings, of course, especially for those who had been pursuing Kaluza-Klein supergravity prior to the 1984 superstring revolution, was the dimensionality of spacetime. If supersymmetry permits eleven spacetime dimensions, why should the theory of everything stop at ten? Richard Feynman always used to say that, in physics, whatever is not forbidden is compulsory.

* *The membrane problem*

This problem rose to the surface again in 1987 when Eric Bergshoeff of the University of Groningen, Ergin Sezgin, now at Texas A&M University, and Paul Townsend from the University of Cambridge discovered *The eleven-dimensional supermembrane*. This membrane has two spatial dimensions and moves in a spacetime dictated by our old friend: eleven-dimensional supergravity! It can either be

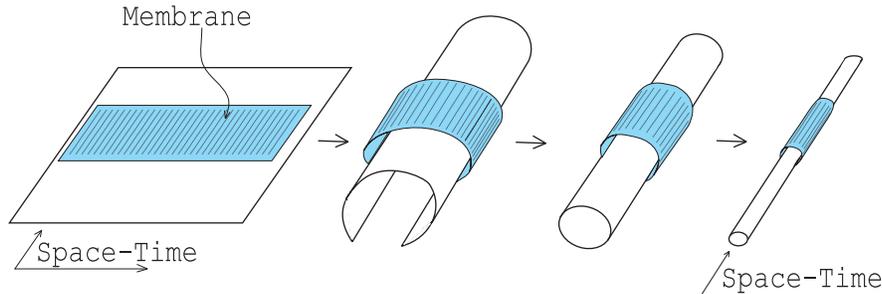


Figure 13: A ten-dimensional string interpreted as an eleven-dimensional membrane wrapped around the extra dimension.

a bubble-like extended object or an infinite sheet. In the same year, moreover, Paul Howe (King’s College, London University), Takeo Inami (Kyoto University), Kellogg Stelle (Imperial College) and I were then able to show that if one of the eleven dimensions is a circle, then we can wrap one of the membrane dimensions around it so that, if the radius of the circle is sufficiently small, it looks like a string in ten dimensions. See Figure 13. In fact, it yields precisely the Type *IIA* superstring. This suggested to us that maybe the eleven-dimensional theory was the more fundamental after all, though this was not a popular view among the latter-day *Flatlanders* who refused to acknowledge this extra dimension^h.

10 Supermembranes

Membrane theory has a strange history which goes back even further than strings. The idea that the elementary particles might correspond to modes of a vibrating membrane was put forward originally in 1960 by the British Nobel Prize winning physicist Paul Dirac, a giant of twentieth century science who was also responsible for two other daring postulates: the existence of *anti-matter* and the existence of *magnetic monopoles*. (Anti-particles carry the same mass but opposite charge from particles and were discovered experimentally in the 1930s. Magnetic monopoles carry a single magnetic charge and to this day have not yet been observed. Interestingly enough, however, they provide an alternative reason why electric charge should be quantized. Once again,

$$e_n = ne, \tag{12}$$

just as Klein had said, but according to Dirac the fundamental charge e is now given by

$$e = 2\pi/g \tag{13}$$

where g is the magnetic charge. This later gave rise to speculations that Nature might be invariant under an electric-magnetic duality that exchanges electric par-

^hSee Appendix.

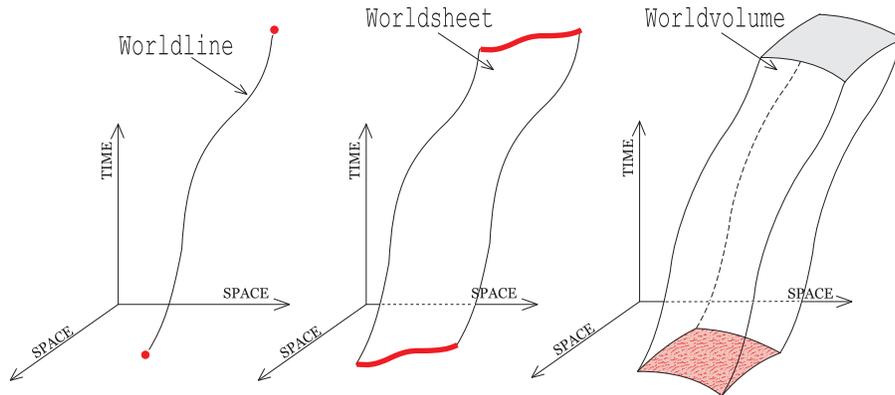


Figure 14: Particles, strings and membranes.

ticles and magnetic monopoles. As we shall see, both the Klein rule and the Dirac rule feature prominently in M -theory, as does electric-magnetic duality.)

When string theory came along in the 1970s, there were some attempts to revive Dirac's membrane idea but without much success. The breakthrough did not come until 1986 when James Hughes, Jun Liu and Joseph Polchinski of the University of Texas showed that, contrary to the expectations of certain string theorists, it was possible to combine the membrane idea with supersymmetry: the *supermembrane* was born.

Consequently, while all the progress in superstring theory was being made a small but enthusiastic group of theorists were posing a seemingly very different question: Once you have given up 0-dimensional particles in favor of 1-dimensional strings, why not 2-dimensional membranes or in general p -dimensional objects (inevitably dubbed p -branes)? Just as a 0-dimensional particle sweeps out a 1-dimensional *worldline* as it evolves in time, so a 1-dimensional string sweeps out a 2-dimensional *worldsheet* and a p -brane sweeps out a d -dimensional *worldvolume*, where $d = p + 1$. See Figure 14.

Of course, there must be enough room for the p -brane to move about in spacetime, so d must be less than the number of spacetime dimensions D . In fact supersymmetry places further severe restrictions both on the dimension of the extended object and the dimension of spacetime in which it lives. One can represent these as points on a graph where we plot spacetime dimension D vertically and the p -brane dimension $d = p + 1$ horizontally. This graph is called the *brane-scan*. See Figure 15. Curiously enough, the maximum spacetime dimension permitted is eleven, where Bergshoeff, Sezgin and Townsend found their 2-brane. In the early 80s Green and Schwarz had showed that spacetime supersymmetry allows classical superstrings moving in spacetime dimensions 3, 4, 6 and 10. (Quantum considerations rule out all but the ten-dimensional case as being truly fundamental. Of course some of these ten dimensions could be curled up to a very tiny size in the way suggested by Kaluza and Klein. Ideally six would be compactified in this way so as to yield the four spacetime dimensions with which we are familiar.) It was

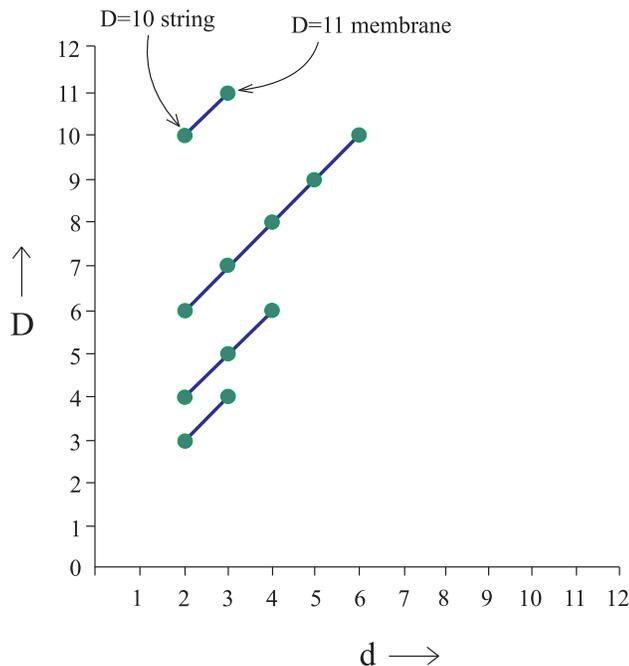


Figure 15: The branescan, circa 1987.

now realized, however, that there were twelve points on the scan which fall into four sequences ending with the superstrings or 1-branes in $D = 3, 4, 6$ and 10 , which were now viewed as but special cases of this more general class of supersymmetric extended object.

Other branes were later added to the scan. For example, in 1991, after Stelle and I had shown how the electric 2-brane emerges as a solution of $D = 11$ supergravity, Rahmi Guven, from University of the Bosphorus, discovered an eleven-dimensional super 5-brane which acts as its magnetic dual.

Notwithstanding these and subsequent results, the supermembrane enterprise was ignored by most adherents of conventional superstring theoryⁱ. Those who had worked on eleven-dimensional supergravity and then on supermembranes spent the early eighties arguing for *spacetime* dimensions greater than four, and the late eighties and early nineties arguing for *worldvolume* dimensions greater than two. The latter struggle was by far the more bitter^j.

ⁱA notable exception was Paul Townsend who continued to keep the faith and whose 1994 paper with Christopher Hull of Queen Mary College, London, did much to convert the sceptics.

^jOne string theorist I know would literally cover up his ears whenever the word “membrane” was mentioned within his earshot! Indeed, I used to chide my more conservative string theory colleagues by accusing them of being unable to utter the M-word.

11 M theory and duality

The uniqueness problem and the dimension problem were suddenly solved simultaneously by Witten in his, by now famous, talk at the University of Southern California in February 1995. Witten put forward a convincing case that the distinction we used to draw between the five consistent string theories was merely an artifact of our approximation scheme and that when looked at exactly, there was really only one theory, which subsumed all the others. Moreover this theory was a supersymmetric theory in eleven dimensions! In fact, when viewed at distances much larger than the Planck length, it is approximated by our old friend eleven-dimensional supergravity!

Curiously enough, however, Witten still played down the importance of supermembranes. But it was only a matter of time before he too succumbed to the conclusion that we weren't doing just string theory any more! In the coming months, literally hundreds of papers appeared on the internet confirming that, whatever this eleven-dimensional theory may be, it certainly involves supermembranes in an important way. Consequently Witten dubbed it *M-theory* “where M stands for Magic, Mystery or Membrane according to taste”.

Even the chiral $E_8 \times E_8$ string, which according to Witten's earlier theorem could never come from eleven-dimensions, was given an eleven-dimensional explanation by Petr Horava (Princeton University) and Witten. The no-go theorem is evaded by compactifying not on a circle (which has no ends), but on a line-segment (which has two ends). It is ironic that having driven the last nail into the coffin of eleven-dimensions (and having driven Gell-Mann to utter “Ugh!”), Witten was the one to pull the nail out again! He went on to argue that if the size of this one-dimensional space is large compared to the six-dimensional space, then our world is approximately five-dimensional. This may have important consequences for confronting *M-theory* with experiment.

What do we now know with M-theory that we did not know with old-fashioned string theory? Here are a few examples, references to which may be found in [4].

1) Electric-magnetic duality in $D = 4$ is a consequence of membrane/fivebrane duality in $D = 11$. Moreover, the Klein charge quantization rule and the Dirac charge quantization rule play dual roles. They are thus complementary rather than contradictory.

2) *Exact* electric-magnetic duality, first proposed for maximally supersymmetric quark-gluon theories, has been extended to *effective* duality by Nathan Seiberg (Princeton) and Witten to less supersymmetric theories: the so-called Seiberg-Witten theory. This has been very successful in providing the first proofs of quark confinement (albeit in the as-yet-unphysical super QCD) and in generating new pure mathematics on the topology of four-manifolds. Seiberg-Witten theory and other dualities of Seiberg may, in their turn, be derived from M-theory.

3) Indeed, it seems likely that all supersymmetric quantum field theories with any symmetry, and their spontaneous symmetry breaking, admit a geometrical interpretation within M-theory as the worldvolume fields that propagate on the common intersection of stacks of p-branes wrapped around various holes of the

compactified dimensions, with the Higgs field given by the brane separations.

4) In string theory, the vacuum degeneracy problem arises because there are billions of Calabi-Yau vacua which are distinct according to classical topology. Like higher-dimensional Swiss cheeses, each can have different number of p -dimensional holes. This results in many different kinds of four-dimensional Standard-like Models with different symmetries, numbers of families and different choices of quarks and leptons. Moreover, M-theory introduces new effects which allow many more possibilities, making the degeneracy problem apparently even worse. However, most (if not all) of these manifolds are in fact smoothly connected in M-theory by shrinking the p -branes that can wrap around the p -dimensional holes in the manifold and which appear as black holes in spacetime. As the wrapped-brane volume shrinks to zero, the black holes become massless and effect a smooth transition from one Calabi-Yau manifold to another. Although this does not yet cure the vacuum degeneracy problem, it puts it in a different light. The question is no longer why we live in one topology rather than another but why we live in one particular corner of the unique topology. This may well have a simpler explanation. A fuller discussion may be found in Brian Greene's book [1].

Another interconnection was uncovered by Polchinski who realized that the Type II super p -branes carrying a certain kind of charge may be identified with the so-called Dirichlet-branes (or D -branes, for short) that he had studied some years ago by looking at strings with unusual boundary conditions. Dirichlet was a French mathematician who first introduced such boundary conditions. These D -branes are just the surfaces on which open strings can end. This D -brane technology has opened up a whole new chapter in the history of supermembranes.

5) Thus another by-product of these membrane breakthroughs has been an appreciation of the role played by black holes in particle physics and string theory. In fact they can be regarded as black branes wrapped around the compactified dimensions. These black holes are tiny (10^{-35} meters) objects; not the multi-million solar mass objects that are gobbling up galaxies. However, the same physics applies to both and there are strong hints that M-theory may even clear up many of the apparent paradoxes of quantum black holes raised by Stephen Hawking in Cambridge. Ever since the 1970's, when Hawking used macroscopic arguments to predict that black holes have an entropy equal to one quarter the area of their event horizon, a microscopic explanation has been lacking. But treating black holes as wrapped p -branes, together with the realization that Type II branes have a dual interpretation as Dirichlet branes, allowed Andrew Strominger and Cumrun Vafa of Harvard to make the first microscopic prediction in complete agreement with Hawking. The fact that M-theory is clearing up some long standing problems in quantum gravity gives us confidence that we are on the right track.

6) It is known that the strengths of the four forces change with distance. In supersymmetric extensions of the Standard Model, one finds that the strengths of the strong, weak and electromagnetic forces all meet at about at very short distances entirely consistent with the idea of Grand Unification. The strength of the gravitational force also almost meets the other three, but not quite. This near miss has been a source of great interest, but also frustration. However, in a

universe of the kind envisioned by Witten, spacetime is approximately a narrow five dimensional layer bounded by four-dimensional walls^k. The particles of the Standard Model live on the walls but gravity lives in the five-dimensional bulk. As a result, it is possible to choose the size of this fifth dimension so that all four forces meet at this common scale. Note that this is much larger than the Planck length, so gravitational effects may be much closer than we previously thought; a result that would have all kinds of cosmological consequences. Indeed, this has led to a revival of a variation on the Kaluza-Klein theme whereby our universe is a 3-brane embedded in a higher-dimensional spacetime. Thus the strong, weak and electromagnetic forces might be confined to the worldvolume of the brane while gravity propagates in the bulk. It has recently been suggested that, in such schemes, the extra dimension might be much larger than 10^{-35} meters and may even be as large as a millimeter [17]. In yet another variation due to Lisa Randall of Harvard and Raman Sundrum of Johns Hopkins, the extra fifth dimension is infinite.

Thus branes are no longer the ugly-ducklings of string theory, but now play center stage in theoretical physics: as the microscopic constituents of M-theory, as the higher-dimensional progenitors of black holes and as entire universes in their own right.

12 So what is *M*-theory?

So are we quarks, strings, branes or what?

New York Times, Tuesday September 22, 1998

I hope to have convinced you that the answer involves “branes or what”, but there is still no final answer to the exact definition of M-theory, although several different proposals have been made. Is M-theory to be regarded literally as membrane theory? In other words should we attempt to “quantize” the eleven dimensional membrane in some, as yet unknown way? Personally, I think the jury is still out on whether this is the right thing to do.

Tom Banks and Stephen Shenker at Rutgers together with Willy Fischler from the University of Texas and Lenny Susskind at Stanford have even proposed a rigorous definition of *M*-theory known as M(atrrix) theory which is based on an infinite number of Dirichlet 0-branes. In this picture spacetime is a fuzzy concept in which the spacetime coordinates x, y, z, \dots are matrices that do not commute e.g. $xy \neq yx$. This approach has generated great excitement but does yet seem to be

^kIf this picture is correct, we really need only five dimensions with the other six going along for the ride. Why should Nature behave like this? The only good answer to this question I could find is in Mother Goose’s Nursery Rhymes:

*Nature requires five,
 Custom allows seven,
 Idleness takes Nine,
 And Wickedness Eleven.*

the last word. It works well in high dimensions but as we descend in dimension it seems to break down before we reach the real four-dimensional world.

Another interesting development has been provided by Juan Maldacena at Princeton, who has suggested that M -theory compactified to a particular D -dimensional spacetime, including all its gravitational interactions, may be completely described by a non-gravitational theory that resides on the $(D - 1)$ -dimensional boundary of that spacetime. This holds promise not only of a deeper understanding of M -theory, but may also throw light on aspects of the theories that live on the boundary, which in some circumstances can include the kinds of 4-dimensional quark theories that govern the strong nuclear interactions such as QCD. Many theorists are understandably excited about this correspondence because of what it can teach us about QCD. In my opinion, however, this is, in a sense, a diversion from the really fundamental question: what is M -theory? So my hope is that Maldacena's idea will be a two-way process and that it will also teach us more about M -theory.

M -theory has sometimes been called the *Second Superstring Revolution*, but I feel this is really a misnomer. It certainly involves new ideas every bit as significant as those of the 1984 string revolution, but its reliance upon supermembranes and eleven dimensions makes it sufficiently different from traditional string theory to warrant its own name. One cannot deny the tremendous historical influence of superstrings on our current perspectives. Indeed, it is the pillar upon which our belief in a quantum consistent M -theory rests. In my opinion, however, the focus on one-dimensional objects moving in a ten-dimensional spacetime, that prevailed during the period 1984-1995, will ultimately be seen to be a small corner of M -theory [3]. Indeed, future historians may well judge this era as a time when theorists were like boys playing by the sea shore, and diverting themselves with the smoother pebbles or prettier shells of ten-dimensional superstrings while the great ocean of eleven-dimensional M -theory lay all undiscovered before them.

13 Acknowledgments

I would like to thank Dean Shirley Neuman and the University of Michigan Administration for conferring on me the title of Oskar Klein Collegiate Professor of Physics. It is indeed a great honor. I would also like to thank the Physics Department Chair, Ctirad Uher for recruiting me to Michigan in 1999; without him I would not be here. What impressed me most about the University was the intellectual caliber of the faculty and I would also like to acknowledge my colleagues in the Physics Department for providing such a stimulating environment in which to conduct research. Thanks to Stanley Deser and Jim Liu for suggesting improvements in the paper and to Alex Batrachenko for invaluable help with the pictures and diagrams. Living with a physicist is not easy. So (and at the risk of this sounding more like the Oscars than the Oskars), I would also like to thank my wife Lesley, my daughter Jessica and my son Matthew, for putting up with me.

A Flatland, Modulo 8

A.1 Modern Translation

Edwin A. Abbott (1838-1926) was a English clergyman and Shakespearean scholar who wrote his satire, *Flatland: A Romance of Many Dimensions* [18], in 1884. Hence it is first necessary to provide a dictionary of updated terminology:

2-dimensional Flatland. 10-dimensional String Theory
Line-like objects. Strings
A. Square. A Humble String Theorist, Hero of Our Story
His sons, the Pentagons. His Postdocs
His grandsons, the Hexagons. His Graduate Students
Polygons. Full Professors
Circles. The Sultans of String
High Council. Recipients of the MacArthur “Genius” Awards in String Theory
President of the Council. Edgar Whittington, Fields Medalist
The Secret Archives. arXiv.org
The Prefect. Jaguar Quark-Mann, Nobel Laureate
The Palace of the Prefect. California Institute of Technology
3-dimensional Spaceland. 11-dimensional M-theory
Space-like objects. Membranes
Sphere. An M-theorist
Brightness. Electric Charge, or Eleventh Component of Momentum
Height. Radius of the Eleventh Dimension
Policemen. Journalists
Land of 4-dimensions¹. 12-dimensional F-theory
Point-like objects. Particles
King of Pointland. Sheldonian Glasgow, Nobel Laureate
The Abyss of No Dimensions. Quantum Field Theory

The story has been shortened but, apart from the above updates, the words below are exactly as Abbott wrote them. (In the first part of the book, he lampoons the attitude to women in Victorian society, especially in Science. Readers are invited to update that part.) For dramatic effect, Abbott sets the scene on New Year’s Eve, 1999. Although this was remarkably prescient, historians of science now believe that these events actually took place ten years earlier. Now read on.

A.2 Concerning a stranger from the Eleventh Dimension

It was the last day of the 1999th year of our era. The pattering of the rain had long ago announced nightfall; and I was sitting in the company of my wife, musing

¹A speculative idea, whose status is still obscure at the end of the story.

on the events of the past and the prospects of the coming year, the coming century, the coming Millennium.

I was rapt in thought, pondering in my mind some words that had casually issued from the mouth of my youngest graduate student, a most promising young man of unusual brilliancy. I had been induced to reward him by giving him a few hints on Arithmetic, as applied to String Theory.

“I suppose an Eleventh Dimension must mean something in String Theory; what does it mean?” “Nothing at all,” replied I, “not at least in String Theory; for String Theory has only Ten Dimensions.” “Go,” said I, a little ruffled by this interruption; “if you would talk less nonsense, you would remember more sense.”

So my graduate student disappeared in disgrace; and I sat by my wife’s side, endeavouring to form a retrospect of the year 1999 and of the possibilities of the year 2000, but not quite able to shake off the thoughts suggested by the prattle of my bright little graduate student. Only a few sands now remained in the half-hour glass.

Straightway, I became conscious of a Presence in the room, and a chilling breath thrilled through my very being.

“The boy is a fool, I say; Eleven Dimensions can have no meaning in String Theory.” At once there came a distinctly audible reply, “The boy is not a fool; and Eleven Dimensions has an obvious Geometrical meaning.” What was my horror when I saw before me a Figure! I should have thought it a String Theorist only that it seemed to change its size in a manner impossible for a String Theorist or for any regular Figure of which I had had experience.

“I am, in a certain sense a String Theorist,” replied the Voice, “and a more perfect String Theorist than any in Ten Dimensions; but to speak more accurately, I am an M-theorist.”

I glanced at the half-hour glass. The last sands had fallen. The third Millennium had begun.

A.3 How the stranger vainly endeavoured to reveal to me in words the mysteries of Eleven Dimensional M-theory

I: Would your Lordship indicate or explain to me in what direction is the Eleventh Dimension, unknown to me.

Stranger: I came from it. It is up above and down below.

I: Your Lordship jests.

Stranger: I am in no jesting humour. I tell you that I come from M-theory, or, since you will not understand what M-theory means, from the Land of Membranes whence I lately looked down upon your Plane that you call String forsooth. From that position of advantage I discerned all that you speak of as a Membrane.

I: Such assertions are easily made, my Lord.

Stranger: But not easily proved, you mean. But I mean to prove mine.

I: But am I to suppose that your Lordship gives to Electric Charge the title of Dimension, and that what we call “Charge” you call eleventh component of momentum?

Stranger: No, indeed. By the eleventh coordinate, I mean a dimension like your length: only, with you, the radius of the eleventh dimension is not so easily perceptible, being extremely small. For even an M-theorist-which is my proper name in my own country- if he manifest himself at all to an inhabitant of Ten Dimensions-must needs manifest himself as a String Theorist.

Every reader in M-theory will easily understand that my mysterious Guest was speaking the language of truth and even simplicity. But to me, proficient though I was in String Mathematics, it was by no means a simple matter.

“Monster,” I shrieked, “be thou juggler, enchanter, dream or devil, no more will I endure thy mockeries. Either thou or I must perish.” And saying these words I precipitated myself upon him.

A.4 How the M-theorist, having in vain tried words, resorted to deeds

M-theorist: Why do you refuse to listen to reason? I had hoped to find in you-as being a man of sense and an accomplished mathematician- a fit apostle for the Gospel of the Eleven Dimensions, which I am allowed to preach once only in a thousand years: but now I know not how to convince you. Away from me, or you must go with me- whither you know not-into the Land of Eleven Dimensions!

“Fool! Madman!” I exclaimed.

“Ha! Is it come to this?” thundered the Stranger: “then meet your fate: out of your Plane you go. Once, twice, thrice! ‘Tis done!”

A.5 How I came to the Eleventh Dimension and what I saw

An unspeakable horror seized me. There was a darkness; then a dizzy sickening sensation of sight that was not like seeing. When I could find voice, I shrieked aloud in agony, “Either this is madness or it is Hell.” It is neither replied the voice of the M-theorist, “it is Knowledge; it is Eleven Dimensions: open your eyes once again and try to look steadily.”

I looked, and behold, a new world! Something-for which I had no words; but you, my readers in M-theory, would call a Membrane.

Bewildered though I was by my Teacher’s enigmatic utterance, I no longer chafed against it, but worshipped him in silent adoration. He continued with more mildness in his voice. “Distress not yourself if you cannot at first understand the deep mysteries of Eleven Dimensions. By degrees they will dawn upon you. But enough of this. Look yonder. Do you know that building?”

I looked, and afar off I saw an immense Polygonal structure, in which I recognised the General Assembly Hall of the States of String Theory; and I perceived

that I was approaching the great Metropolis.

It was now morning, the first hour of the first day of the two thousandth year of our era. Acting, as was their wont, in strict accordance with precedent, the highest String Theorists of the realm were meeting in solemn conclave^m.

The minutes of the previous meeting were now read: “Whereas the States had been troubled by divers ill-intentioned persons pretending to have received revelations from another World, and professing to produce demonstrations whereby they had instigated to frenzy both themselves and others, it had been for this cause unanimously resolved by the MacArthur Fellows that on the first day of each millenary, special injunctions be sent to the Prefects in several districts of String Theory, to make strict search for such misguided persons and without formality of mathematical examination, to destroy all such..”

“You hear your fate”, said the M-theorist to me, while the MacArthur Fellows were passing for the third time the formal resolution. “Death or imprisonment awaits the Apostle of the Eleven Dimensions”. “Not so, replied I, “the matter is now so clear to me, the nature of real space so palpable, that methinks I could make a child understand it. Permit me but to descend at this moment and enlighten them.” “Not yet”, said my Guide, “the time will come for that. Meantime I must perform my mission. Stay thou there in thy place.” Saying these words, he leaped with great dexterity into the sea (if I may so call it) of String Theory, right in the midst of the ring of MacArthur Fellows. “I come,” cried he, “to proclaim that there is a land of Eleven Dimensions.”

I could see many of the younger Fellows start back in manifest horror.

“My Lords,” said President Edgar Whittington to the Junior String Theorists, “there is not the slightest need for surprise; the secret archives, to which I alone have access, tell me that a similar occurrence happened on the last two millennial commencementsⁿ. You will, of course, say nothing of these trifles outside the Cabinet.”

Raising his voice, he now summoned the guards. “Arrest the journalists; gag them. You know your duty.” After he had consigned to their fate the wretched journalists-ill-fated and unwilling witnesses of a State-secret which they were not permitted to reveal-he again addressed the String Theorists. “My Lords, the business of the Fellows being concluded, I have only to wish you happy New Year.”

^mAbbott can only be referring to the *Strings 2000* conference, but historians favor *Strings 1990*, whose Organizing Committee is known to have passed the following resolution (with one dissenting vote): “The word *Membrane* shall nowhere appear in the topics listed on the *Strings 1990* conference poster.”

ⁿSome historians place these secret archives at Los Alamos National Laboratory, but they now seem to have disappeared.

A.6 How, though the M-theorist shewed me other mysteries of Eleven Dimensions, I still desired more; and what came of it

Once more we ascended into Eleven Dimensions. “Hitherto,” said the M-theorist, “I have shewn you naught save Branes. Now I must introduce you to Stacks, and reveal to you the plan upon which they are constructed. Behold this multitude of square cards. See, I put one upon the other, not as you supposed, Northward of the other, but *on* the other. Now a second, now a third. See, I am building up a Stack by a multitude of Branes parallel to one another. Now the Stack is complete.

Were I to give the M-theorist’s explanation of these matters, succinct and clear though it was, it would be tedious to an inhabitant of Eleven Dimensions, who knows these things already. Suffice it that I could now readily distinguish between a String and a Membrane.

This was the Climax, the Paradise, of my strange eventful history. Henceforth I have to relate the story of my miserable Fall: most miserable, yet surely most undeserved! For why should the thirst for knowledge be aroused, only to be disappointed and punished? My volition shrinks from the painful task of recalling my humiliations; yet, like a second Prometheus, I will endure this and worse, if by any means I may arouse in the interiors of String Humanity a spirit of rebellion against the Conceit which would limit our Dimensions to Ten or even Eleven.

I: My Lord, your own wisdom has taught me to aspire to One even more great, more beautiful, and more closely approximate to Perfection than yourself. As you yourself, superior to all Ten Dimensional forms, combine many strings in One, so doubtless there is One above you who combines many Membranes in One Supreme Existence, surpassing even M-theory. And even as we, who are now in Eleven Dimensions, look down on Ten Dimensions, so of a certainty there is yet above us some higher, purer F-theory - some yet more spacious Space, some more dimensionable Dimensionality.

M-theorist: Pooh! Stuff! Enough of this trifling! Time is short, and much remains to be done before you are fit to proclaim the Gospel of Eleven Dimensions to your blind benighted countrymen in String Theory.

I: What therefore more easy than now to take his servant on a second journey into the blessed region of the Twelfth Dimension?

M-theorist: But where is this land of Twelve Dimensions?

I: I know not: but doubtless my Teacher knows.

M-theorist: Not I. There is no such land. The very idea of it is utterly inconceivable.

My words were cut short. Down! Down! Down! I was rapidly descending; and I knew that a return to String Theory was my doom.

A.7 How the M-theorist encouraged me in a vision

When I was at last by myself, a drowsy sensation fell on me; but before my eyes closed I endeavoured to reproduce the Eleventh Dimension. During my slumber I

had a dream.

“Look yonder,” said my guide, “in String Theory thou hast lived; thou hast soared with me to the heights of M-theory; now, in order to complete the range of thy experience, I conduct thee downward to the lowest depth of existence, even to Quantum Field Theory, the Abyss of No Dimensions.

“Behold yon miserable creature. That Particle Physicist, Sheldonian Glasgow, is a Being like ourselves, but confined to the non-dimensional gulf. He is himself his own World, his own Universe; of any other than himself he can form no conception; he knows neither String nor Membrane, for he has no experience of them. Yet mark his perfect self-contentment.

“Can you not startle the little thing out of its complacency?” said I. “Tell it what it really is, as you told me; reveal to it the narrow limitations of Pointland, and lead it up to something higher.” “That is no easy task,” said my master. “Let us leave this King of Pointland to the ignorant fruition of his omnipresence and omniscience: nothing that you or I can do can rescue him from his self-satisfaction.”

After this, as we floated gently back to Ten Dimensional String Theory, I could hear the mild voice of my Companion pointing the moral of my vision, and stimulating me to aspire, and to teach others to aspire.

A.8 How I tried to teach the theory of Eleven Dimensions to my graduate student, and with what success

I awoke rejoicing, and began to reflect on the glorious career before me. I would go forth, methought, at once, and evangelize the whole of String Theory.

Just as I had decided on the plan of my operations, I heard a loud voice. It was the a herald’s proclamation. Listening attentively, I recognized the words of the Resolution of the MacArthur Fellows, enjoining the arrest, imprisonment, or execution of any one who should pervert the minds of the people by delusions, and by professing to have received revelations from another World.

I reflected. This danger was not to be trifled with. It would be better to avoid it by omitting all mention of my Revelation.

My Postdocs were men of character and standing, and physicists of no mean reputation, but not great in mathematics, and, in that respect, unfit for my purpose. But it occurred to me that my young docile graduate student, with his mathematical turn, would be a most suitable pupil. Discussing the matter with him, a mere boy, I should be in perfect safety; for he would know nothing of the proclamation of the MacArthur Fellows; whereas I could not feel sure that my Postdocs- so greatly did their patriotism and reverence for the Sultans of String predominate over mere blind affection-might not feel compelled to hand me over to the Prefect, Jaguar Quark-Mann, if they found me seriously maintaining the seditious heresy of the Eleventh Dimension.

At this moment we heard once more the herald’s “Oh Yes! Oh Yes!” outside in the street proclaiming the resolution of the MacArthur Fellows. Young though he was, my graduate student-who was unusually intelligent for his age, and bred up in perfect reverence for the Sultans of String-took in the situation with an acuteness

for which I was quite unprepared. He remained silent till the last words of the proclamation had died away, and then, bursting into tears, "Dear Square," he said, "that talking of Eleven Dimensions was only my fun, and of course I meant nothing at all by it; and we did not know anything then about the new Law; and I don't think I said anything about M-theory. How silly it is! Ha! Ha! Ha!"

Thus ended my first attempt to convert a pupil to the Gospel of Eleven Dimensions.

A.9 How I then tried to diffuse the theory of Eleven Dimensions by other means, and of the result

So I devoted several months in privacy to the composition of a treatise on the mysteries of Eleven Dimensions. Only, with a view of evading the Law, if possible, I spoke not of a physical Dimension, but of Thoughtland.

Meanwhile my life was under a cloud. All pleasures palled upon me; all sights tantalized and tempted me to outspoken treason, because I could not but compare what I saw in Ten Dimensions with what it really was if seen in Eleven, and could hardly refrain from making my comparisons aloud.

I felt that I would have been willing to sacrifice my life for the Cause, if thereby I could have produced conviction. But if I could not convince my graduate student, how could I convince the highest and most developed String Theorists in the land?

And yet at times my spirit was too strong for me, and I gave vent to dangerous utterances. Already I was considered heterodox if not treasonable, and I was keenly alive to the danger of my position; nevertheless I could not at times refrain from bursting out into suspicious or half-seditious utterances, even among the highest Full Professors and most developed Sultans of String society.

At last, to complete a series of minor indiscretions, at a meeting of our Local Speculative Society held at the Palace of the Prefect, California Institute of Technology,- some extremely silly person having read an elaborate paper exhibiting the precise reasons why Providence has limited the number of Dimensions to Ten-I so far forgot myself as to give an exact account of the whole of my voyage with the M-theorist into Eleven Dimensions, and to the Assembly Hall in our Metropolis, and then to Eleven Dimensions again, and of my return home, and of everything that I had seen and heard in fact or vision. At first, indeed, I pretended that I was describing the imaginary experiences of a fictitious person; but my enthusiasm soon forced me to throw off all disguise, and finally, in a fervent peroration, I exhorted all my hearers to divest themselves of prejudice and to become believers in the Eleventh Dimension.

Need I say that I was at once arrested and taken before the MacArthur Fellows?

After I had concluded my defence, President Whittington, perhaps perceiving that some of the Junior Fellows had been moved by my evident earnestness, asked me two questions:-

1. Whether I could indicate the direction which I meant when I used the word *Eleven*?

2. Whether I could by any diagrams indicate the figure I was pleased to call a Membrane?

I declared that I could say nothing more, and that I must commit myself to the Truth, whose cause would surely prevail in the end.

The President replied that I must be sentenced to perpetual imprisonment; but if the Truth intended that I should emerge from prison and evangelize the world, the Truth might be trusted to bring that result to pass.

Here I am, absolutely destitute of converts, and, for aught that I can see, the millennial Revelation has been made to me for nothing. Prometheus was bound for bringing down fire for mortals, but I-poor 10-dimensional String Theory Prometheus-lie here in prison for bringing down nothing to my countrymen. Yet I exist in the hope that these memoirs, in some manner, I know not how, may find their way to the minds of humanity in Some Dimension, and may stir up a race of rebels who shall refuse to be confined to limited Dimensionality.

A.10 Postscript

Although Abbott never wrote about it, the story has a happy ending. Five years later, the President of the Council made an historic announcement at the University of Southern California lifting the ban on Eleven Dimensions. Fearing civil unrest, however, he continued to proscribe discussion of Membranes. After the String Theorists were given time to get over the shock of Eleven Dimensions, though, even Membranes ceased to be taboo. The gag on the journalists was removed, and our hero the Square was released from prison. Indeed, he has since been promoted to a Polygon.

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