# Thirty years of Erice on the brane<sup>1</sup>

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#### Abstract

After initially meeting with fierce resistance, *branes*, p-dimensional extended objects which go beyond particles (p=0) and strings (p=1), now occupy centre stage in theoretical physics as microscopic components of M-theory, as the seeds of the AdS/CFT correspondence, as a branch of particle phenomenology, as the higher-dimensional progenitors of black holes and, via the *brane-world*, as entire universes in their own right. Notwithstanding this early opposition, Nino Zichichi invited me to to talk about supermembranes and eleven dimensions at the 1987 School on Subnuclear Physics and has continued to keep Erice on the brane ever since. Here I provide a distillation of my Erice brane lectures and some personal recollections.

<sup>&</sup>lt;sup>1</sup>Based on lectures at the International Schools of Subnuclear Physics 1987-2017 and the International Symposium 60 Years of Subnuclear Physics at Bologna, University of Bologna, November 2018.

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## 13 Acknowledgements

So are we quarks, strings, branes or what?

New York Times, September 22, 1998

# 1 Introduction

### 1.1 Geneva and Erice: a tale of two cities

In 1987 I was a staff member in the Theory Division at CERN, on leave of absence from Imperial College London. I spent the early 1980s advocating spacetime dimensions greater than four [43] and the late 1980s advocating worldvolume dimensions greater than two [150]. The latter struggle was by far the harder. See for example [352]. At this time CERN was playing a prominent part in the development of branes and the 11-dimensional foundations of what was later to be called M-theory. See, for example, CERN TH-4124-85 [31], CERN-TH-4664-87 [50], CERN-TH-4731-87 [51], CERN-TH-4749-87 [52], CERN-TH-4779-87 [53], CERN-TH-4797-87 [54], CERN-TH-4818-87 [56], CERN-TH-4820/87 [57], CERN-TH-4924/87 [64]. As a matter of fact, the Oxford English Dictionary attributes first usage of the word *brane* to the May 1987 CERN preprint [51] by Duff, Inami, Pope, Sezgin and Stelle, published the following year in Nuclear Physics B<sup>2</sup>. See Fig 2. Since then, according to INSPIRE there have been 46,192 papers on branes garnering 1,786,998 citations as of November 2018. According to [369], *brane* ranks 13th in the list of most frequent words in hep-th titles<sup>3</sup>.

The 1987 Annual Report of the CERN Theory Division was upbeat:

Finally there were a few papers that are highly critical of string theory and its prospects, and a few that started a heroic study of more complicated objects, namely supermembranes. During 1987 the CERN theory group became the leading research centre for this subject, which is still in its infancy. The main goal is to understand why there exists an elegant and unique eleven dimensional supergravity, while string theory seems to be restricted to ten dimensions.

That year I also co-authored an article for New Scientist with Christine Sutton, former editor of the CERN Courier, entitled *The Membrane at the End of the Universe*,

<sup>&</sup>lt;sup>2</sup>Paul Townsend's lecture at the Trieste Spring School in April 1987 was intended to be entitled "P-branes for pea-brains", but organizer Ergin Sezgin baulked (at pea-brains, not p-branes).

<sup>&</sup>lt;sup>3</sup>The top 20 are model, theory, black-hole, quantum, gravity, string, susy, solution, field, equation, symmetry, brane, inflation, gauge-theory, system, geometry, sugra, new, generalized.

## brane, *n*.

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Quotations: Show all | Hide all Keywords: On | Off

#### Pronunciation: Brit. /brein/, U.S. /brein/ Etymology: Shortened < MEMBRANE n.

Physics.

An extended object with any given number of dimensions, of which strings in string theory are examples with one dimension. Also with prefixed numbers, or symbols representing numbers, as *2-brane*, *p-brane*. Quot. 1988 is from a paper received for publication earlier (18 May 1987) than quot. 1987 (1 Aug.).

- 1987 *Physics Lett.* B. **198** 441 The extension of the spacetime supersymmetric Green–Schwarz covariant superstring action to *p*-dimensional extended objects (*p*-branes) is possible if and only if the on-shell *p*-dimensional Bose and Fermi degrees of freedom are equal.
- 1988 M. DUFF et al. in *Nucl. Physics* B. **297** 516 We shall be concerned only with extended objects of one time and two space dimensions, i.e. '2branes'... Possible 'p-brane' theories exist whenever there is a closed p + 2 form in superspace.
- 1996 Sci. Amer. Jan. 75/2 He [sc. M. J. Duff] found that a five-dimensional membrane, or a 'five-brane', that moved through a 10-dimensional space could serve as an alternative description of string theory.
- 1997 New Scientist 18 Jan. 35/2 A string is a one-brane, an ordinary membrane like a soap bubble is a two-brane, and so on.
- 1998 Independent on Sunday 19 July (Rev. Suppl.) 56/2 With extra dimensions thrown in, strings turn into membranes.—and, to complicate things further, these membranes are also called 'p-branes', where 'p' is the number of dimensions.
- 2000 Nature 2 Mar. 28/3 One of the key ideas..is that the four-dimensional space-time we observe at everyday scales is actually the evolution in time of a three-brane moving through an ambient space-time of higher dimension.

(Hide quotations)

### Figure 1: Oxford English Dictionary: the word brane

describing conformal field theories arising from branes living on the boundary of antide Sitter space (AdS), a theme later to play a part in the AdS/CFT correspondence [227, 230, 231]. See Fig 2. By the way, I apologized to Mike Green for the caption inserted by New Scientist without my knowledge. Mike reminded me recently that at the 1983 High Energy Physics Conference in Brighton, he and I played a game of crazy golf on the promenade in order to decide whether spacetime had ten or eleven dimensions. I won (the golf that is). My excuse for needing a reminder about the golf was that later that same evening I met my future wife.

In 1988, together with fellow brane enthusiasts Chris Pope and Ergin Sezgin, I accepted an invitation from Dick Arnowitt to take up a faculty position in the Physics Department at Texas A&M and was also lucky enough to have Jianxin Lu assigned to me as a graduate student. He and I were to co-author 20 braney papers. There was less brane activity at CERN<sup>4</sup>, see for example, CERN-TH-4970/88 [75], CERN-TH-5048-88 [76], CERN-TH-5188-88 [80], CERN-TH-5239-88 [80], as reflected in the tone of the Annual Report for 1988:

<sup>&</sup>lt;sup>4</sup>But more Type IIA&M-theory in Texas.



Figure 2: 1987 article in New Scientist

During 1988 string theory has continued to thread much of the formal work in the TH division, renewed attention has been given to the development of conformal field theory and, relative to the previous year, work on supermembranes has somewhat shrunk.

For those wishing to follow its past and present activities, the CERN Theory Division has three recommendations shown in Fig. 3. The two historical references seem

The two following references give some of the history of CERN-TH:

• Physics in the CERN Theory Division by John Iliopoulos, 1993

• Theoretical High Energy Physics #: formal theory by Luis Alvarez-Gaume and Wolfgang Lerche, 2004

A comprehensive snapshot of current research activities in CERN-TH is given by the programme of the 2014 Theory Retreat.

Figure 3: Homepage of the CERN Theory Division http://th-dep.web.cern.ch/cern-theoretical-physics

rather coy about the above activities in supermembranes and we find only

**Physics in the CERN Theory Division**: Strings are the simplest extended objects. Although theories of higher dimensional objects have been studied (membranes, etc.), only strings seem to yield consistent theories.

**Theoretical High Energy Physics**: There was also some activity in the study of the theory of Supermembranes, and in particular in [50] it was shown how to extract 10-dimensional superstrings from 11-dimensional supermembranes.

Whereas branes are at the forefront of current activities:

**Programme of the 2014 Theory Retreat**: includes Brane wrapping 3-cycles, D5-brane effective action, Intersecting 7-branes, D5/M5-brane superpotentials, dSvacua in Type IIB with branes on singularities, Non-perturbative 7-branes, T-branes/gluing branes, Unoriented D-branes, etc etc

Such ambivalence towards branes was not unique to CERN<sup>5</sup>. There are no superstrings in eleven dimensions but there are supermembranes [49, 49, 111] which is why between the 1984 Superstring Revolution and the 1995 M-theory Revolution many string theorists were opposed to eleven dimensions. Membrane-related grant proposals tended to attract hostile referee reports during that period and papers with titles like *Supermembranes: a fond farewell* and *Eleven dimensions (Ugh!)* did not help. One string theorist announced that "I want to cover up my ears every time I hear the word membrane" and some organisers of the annual superstring conferences even banned the use of the M-word. My colleague Paul Townsend, one of the membrane pioneers, compared this with the theatrical superstition of calling Macbeth the M-Play. This opposition continued even after it was shown in 1987 that one of the five consistent ten-dimensional superstring theories, the Type IIA string, was just the limiting case of the eleven-dimensional supermembrane [50]. .

An exception to this negativity was Nino Zichichi and in 1987 he invited me to give two lectures on branes at the School on Subnuclear Physics in Erice. Ironically, an experimentalist could see what many theorists could not: since supermembranes are not forbidden by supersymmetry they must be compulsory. See Section 9.1. He has not only continued to welcome me and others to speak about branes at Erice in the intervening 30 years (together more recently with his co-organizer Gerard 't Hooft) but has also promoted them himself. See [356] for a recent example. I should also mention that another Erice visitor, CERN theorist Sergio Ferrara, was always very supportive [363].

Our purpose here is to give a personal account of these previous lectures and their place in the scheme of things as seen from a 2017 perspective<sup>6</sup>. Accordingly, the Section

<sup>&</sup>lt;sup>5</sup>Later CERN contributions to branes include CERN-TH-6675-93 [134], CERN-TH-7542-94 [150].

<sup>&</sup>lt;sup>6</sup>I have omitted recollections of lectures I gave at the 2006, 2012, 2013, 2014 and 2015 Schools since branes

assigned to each lecture also contains a Subsection devoted to subsequent developments. Of course this means that important topics not anticipated in the lectures will not be discussed as thoroughly as those that were. Other Erice lectures devoted to branes include those of Khuri [115], Witten [156], Polchinski [260], Bachas [268], Antoniadis [283], Randall [296] and Sagnotti [365]. Two other historical accounts which are well worth reading are those of Witten [347] and Polchinski [361].

## 1.2 Co-authors

Thanks to my braney collaborators: Alex Anastasiou, Alex Batrachenko, Eric Bergshoeff, Miles Blencowe, Leron Borsten, Duminda Dahanayake, John Dixon, Sergio Ferrara, Gary Gibbons, Paul Howe, Mia Hughes, Haja Ibrahim, Takeo Inami, Jussi Kalkkinen, Ramzi Khuri, James T. Liu, Hong Lu, Jianxin Lu, Alessio Marrani, Rubin Minasian, Silvia Nagy, Roberto Percacci, Jan Plefka, Chris Pope, Joachim Rahmfeld, William Rubens, Henning Samtleben, Ergin Sezgin, Kelly Stelle, Christine Sutton, Paul Townsend, W. Y. Wen and Edward Witten.

## **1.3** Nomenclature

The names given to various branes have evolved as their place in the scheme of things has become clearer. For example, *M*-theory is an eleven-dimensional unified theory [154, 163] incorporating [347] earlier ideas on duality [122, 133, 138, 152] and on supersymmetric branes [49, 106, 108, 111, 148] that subsumes D=11 supergravity and the five D=10 superstring theories [252]. See Section 11 for the etymology of *M*-theory. Following its discovery, the D=11 supermembrane and super 5-brane became known as the *M2-brane* and *M5-brane* respectively. (Discrete subgroups of) the Cremmer-Julia symmetries [11], conjectured to be brane analogue of string T-dualities [99] became known as *U*-dualities of M-theory [148]. Similarly the Type II p-branes, which appear as CFTs on the boundary of AdS [74] and as closed string solitons carrying Ramond-Ramond charge [107, 106, 108, 124], are now known as *D*-branes following the realization by Polchinski [168] that they admit the dual open string interpretation of *Dirichlet-branes* [87, 88]: surfaces of dimension p on which the open strings can end:

No talk at Texas A&M would be complete without mention of supermembranes. If one compactifies the Type I SO(32) superstring, which is unoriented, and sends  $r \to 0$ ,

were not their main focus.

one obtains a theory with a super-D-brane...

J. Polchinski, Strings 89, Texas A&M, March 1989 [87].

At the same time the heterotic and Type II 5-branes carrying Neveu-Schwarz charge were renamed NS-branes and the fundamental string the F-string to distinguish it from the D-string. The D=6 dyonic string became known as the D1-D5-brane system. In charting the history of these various branes we shall adopt the convention in this lecture of using their modern names unless we are quoting verbatim an earlier lecture. Moreover, we reserve the name D-brane for the 1/2 BPS Type II branes whose mass equals their charge and use the name black branes for those whose mass exceeds their charge.

Just as the scalar multiplet CFT that occupies the boundary of  $AdS_4$  is called the *singleton*, so we call the vector supermultiplet that occupies the boundary of  $AdS_5$  the *doubleton* and the tensor supermultiplet that occupies the boundary of  $AdS_7$  the *tripleton*. This nomenclature is based on the rank of  $AdS_{p+2}$  and differs from [79].

# 2 1987 Not the Standard Superstring Review

1987 INTERNATIONAL SCHOOL OF SUBNUCLEAR PHYSICS - Director: A. ZICHICHI 25th Course: The Super World - II 6 - 14 August 1987 [52]

The first of my lectures at the School on Subnuclear Physics, Not the standard superstring review [52], was an appraisal of the current state of superstrings which differed from the superstring orthodoxy in those heady days following the 1984 Superstring revolution. Specifically I focussed on the vacuum degeneracy problem and supermembranes. However, I tempered my scepticism by saying:

In order not to be misunderstood, let me say straight away that I share the conviction that superstrings are the most exciting development in theoretical physics for many years, and that they offer the best promise to date of achieving the twin goals of a consistent quantum gravity and a unification of all the forces and particles of Nature. Where I differ is the degree of emphasis that I would place on the unresolved problems of superstrings, and the likely time scales involved before superstrings (or something like superstrings) make contact with experimental reality.

## 2.1 Vacuum degeneracy and the multiverse

In the absence of an exhaustive classification, we do not know how many (consistent

compactifications to four-dimensions) there are<sup>7</sup> but it surely runs into billions [55]. For the time being, therefore, the phrase "superstring-inspired phenomenology" can only mean sifting through these billions of heterotic models in the hope of finding one that is realistic. The trouble with this needle-in-a-haystack approach is that even if we found one with good phenomenology, we would be left wondering in what sense this could be called a "prediction" of string theory.

Some cosmologists, on the other hand, accept vacuum degeneracy as a fact of life. They argue that the Universe has billions of different vacua and we just happen to be living in one of them with  $SU(3) \times SU(2) \times U(1)$ , three families etc. In which case, as Murray Gell-Mann puts it, physics will have been reduced to an environmental science like botany.

### 2.2 Supermembranes

Membrane theory has a strange history which goes back even further than strings [279]. The idea that the elementary particles might correspond to modes of a vibrating membrane was put forward originally in 1962 by Dirac [1]. When string theory came along in the 1970s, there were some attempts to revive the membrane idea but things did not change much until 1986 when Hughes, Liu and Polchinski [44] showed that it was possible to combine membranes with supersymmetry: the *supermembrane* was born. Consequently, while all the progress in string theory was going on, a small splinter



Figure 4: Particles, strings and membranes.

group was posing the question: Once you have given up 0-dimensional particles in favor

<sup>&</sup>lt;sup>7</sup>It had already been noted in [43] that there are an infinite number of compact Einstein manifolds in seven dimensions and hence an infinite number of compactifications of D = 11 supergravity down to D = 4.

of 1-dimensional strings, why not 2-dimensional membranes or in general p-dimensional objects (inevitably dubbed p-branes)? Just as a 0-dimensional particle sweeps out a 1-dimensional worldline as it evolves in time, so a 1-dimensional string sweeps out a 2dimensional worldsheet and a p-brane sweeps out a d-dimensional worldvolume, where d = p + 1. See Fig. 4. Of course, there must be enough room for the p-brane to move about in spacetime, so d must be less than or equal to the number of spacetime dimensions D. In fact, as we shall see in Section 3, supersymmetry places further severe restrictions both on the dimension of the extended object and the dimension of spacetime in which it lives [58]. One can represent these as points on a graph where we plot spacetime dimension D vertically and the p-brane dimension d = p+1 horizontally. This graph is called the *brane-scan* [95]. See Table 1. In the early eighties Green and Schwarz [27] had shown that spacetime supersymmetry allows classical superstrings moving in spacetime dimensions 3, 4, 6 and 10. (Quantum considerations rule out all but the ten-dimensional case as being truly fundamental. Of course some of these ten dimensions could be curled up to a very tiny size in the way suggested by Kaluza and Klein [143]. Ideally six would be compactified in this way so as to yield the four spacetime dimensions with which we are familiar.) It was now realized, however, that these 1-branes in D = 3, 4, 6 and 10 should now be viewed as but special cases of this more general class of supersymmetric extended object.

Curiously enough, the maximum spacetime dimension permitted is eleven, where Bergshoeff, Sezgin and Townsend found their supermembrane [49, 70] which couples to eleven-dimensional supergravity [8]. (The 3-form gauge field of D = 11 supergravity had long been suggestive of a membrane interpretation [12]). Moreover, it was then possible to show [50] by simultaneous dimensional reduction of the spacetime and worldvolume that the membrane looks like a string in ten dimensions. In fact, it yields precisely the Type IIA superstring:

We do not yet know whether this "supermembrane" is consistent at the quantum level but the orthodox claim that only strings can be quantum consistent now looks much less certain.

### 2.3 Subsequent developments

• The multiverse

The loss of uniqueness in going from ten dimensions to four, is nowadays called the *Landscape* problem [271, 259]. The many universes are known collectively as the *Multiverse*. See, for example, Linde's A brief history of the multiverse [354], though some might find it too brief.

• M-theory

Branes now play vital role in M-theory. Reviews on branes may be found in [54, 90, 150, 215, 220, 242, 275]. Reviews of *M*-theory may be found in [169, 207, 216, 252, 253, 254, 280].

# 3 1987 From super-spaghetti to super-ravioli

1987 INTERNATIONAL SCHOOL OF SUBNUCLEAR PHYSICS - Director: A. ZICHICHI 25th Course: The Super World - II 6 - 14 August 1987 [95]

Since my second lecture attempted to justify this passage from strings to membranes and bearing in mind its location, I called it *From super-spaghetti<sup>8</sup> to super-ravioli* It began:

Many of the supergravity theories that we used to study a few years ago are now known to be merely the field theory limit of an underlying string theory. For example, N=2a supergravity in 10 dimensions is just the field theory limit of the Type IIA superstring. What are we to make, therefore, of supergravity theories which cannot be obtained from strings such as N = 1 supergravity in eleven dimensions? This is a particularly puzzling example since it is well known that upon dimensional reduction to 10 dimensions, it yields the above-mentioned N = 2a theory. Indeed, if supersymmetry allows  $D \leq 11$ , why do strings stop at D = 10?

## 3.1 The old brane-scan

It is ironic that although one of the motivations for the original supermembrane paper [44] was precisely to find the superthreebrane as a *topological defect* of a supersymmetric field theory in D = 6; the discovery of the other supermembranes proceeded in the opposite direction. Hughes et al. showed that kappa symmetry could be generalized to d > 2 and proceeded to construct a threebrane displaying an explicit D = 6, N = 1 spacetime supersymmetry and kappa invariance on the worldvolume. It was these kappa symmetric Green-Schwarz actions, rather then the soliton interpretation which

<sup>&</sup>lt;sup>8</sup>What better place to recall this than Bologna?

 $D\uparrow$ 11 tss/v v v v v v10s/vvvvv9 ss8 s7st 6 s/vvs/v vvv5ss4 vs/v s/vv3 s/v s/vv2s1 0 23 0 1 4 56  $7 \quad 8 \quad 9 \quad 10 \quad 11 \quad d \rightarrow$ 

Table 1: The old brane-scan involves only scalar multiplets s on the worldvolume; the new one includes vector multiplets v and antisymmetric tensor multiplets t.

was to dominate the early work on the subject<sup>9</sup>. First of all, Bergshoeff, Sezgin and Townsend [49] found corresponding Green-Schwarz actions for other values of d and D, in particular the eleven-dimensional supermembrane.

Let us introduce the coordinates  $Z^M$  of a curved superspace

$$Z^M = (x^\mu, \theta^\alpha) \tag{3.1}$$

and the supervielbein  $E_M{}^A(Z)$  where  $M = \mu, \alpha$  are world indices and  $A = a, \alpha$  are tangent space indices. We also define the pull-back

$$E_i{}^A = \partial_i Z^M E_M{}^A \tag{3.2}$$

We also need the super-*d*-form  $B_{A_d...A_1}(Z)$ . Then the supermembrane action ihas a kinetic term, a worldvolume cosmological term, and a Wess-Zumino term

$$S = T_d \int d^d \xi \left[ -\frac{1}{2} \sqrt{-\gamma} \gamma^{ij} E_i^{\ a} E_j^{\ b} \eta_{ab} + \frac{1}{2} (d-2) \sqrt{-\gamma} + \frac{1}{d!} \epsilon^{i_1 \dots i_d} E_{i_1}^{\ A_1} \dots E_{i_d}^{\ A_d} B_{A_d \dots A_1} \right].$$
(3.3)

<sup>9</sup>Strangely enough in Yau's version of the history, it was the other way around [337]

This action has the virtue that it reduces to the Green-Schwarz superstring action when d = 2.

The target-space symmetries are superdiffeomorphisms, Lorentz invariance and *d*form gauge invariance. The worldvolume symmetries are ordinary diffeomorphisms and kappa invariance referred to earlier which is known to be crucial for superstrings, so let us examine it in more detail. The transformation rules are

$$\delta Z^M E^a{}_M = 0, \quad \delta Z^M E^\alpha{}_M = \kappa^\beta (1+\Gamma)^\alpha{}_\beta \tag{3.4}$$

where  $\kappa^{\beta}(\xi)$  is an anticommuting spacetime spinor but worldvolume scalar, and where

$$\Gamma^{\alpha}{}_{\beta} = \frac{(-1)^{d(d-3)/4}}{d!\sqrt{-\gamma}} \epsilon^{i_1..i_d} E_{i_1}{}^{a_1} E_{i_2}{}^{a_2} \dots E_{i_d}{}^{a_d} \Gamma_{a_{1..a_d}} \quad .$$
(3.5)

Here  $\Gamma_a$  are the Dirac matrices in spacetime and

$$\Gamma_{a_1\dots a_d} = \Gamma_{[a_1\cdots a_d]} \quad . \tag{3.6}$$

This kappa symmetry has the following important consequences:

1) The symmetry is achieved only if certain constraints on the antisymmetric tensor field strength  $F_{MNP.Q}(\mathbf{Z})$  and the supertorsion are satisfied. In particular the Bianchi identity dF = 0 then requires the  $\Gamma$  matrix identity

$$\left(d\bar{\theta}\Gamma_a d\theta\right) \left(d\bar{\theta}\Gamma^{ab_1\dots b_{d-2}} d\theta\right) = 0 \tag{3.7}$$

for a commuting spinor  $d\theta$ . As shown by Achucarro, Evans, Townsend and Wiltshire [58] this is satisfied only for certain values of d and D. Specifically, for  $d \ge 2$ 

$$d = 2: \quad D = 3, 4, 6, 10$$
  

$$d = 3: \quad D = 4, 5, 7, 11$$
  

$$d = 4: \quad D = 6, 8$$
  

$$d = 5: \quad D = 9$$
  

$$d = 6: \quad D = 10 .$$
(3.8)

Note that we recover as a special case the well-known result that Green-Schwarz superstrings exist *classically* only for D = 3, 4, 6, and 10. Note also  $d_{max} = 6$  and  $D_{max} = 11$ . The upper limit of D = 11 is already known in supergravity but there it is necessary to make extra assumptions concerning the absence of consistent higher spin interactions. In this formulation of supermembranes, it follows automatically.

2) The matrix  $\Gamma$  of (1.20) is traceless and satisfies

$$\Gamma^2 = 1 \tag{3.9}$$

when the equations of motion are satisfied and hence the matrices  $(1 \pm \Gamma)/2$  act as projection operators. The transformation rule (1.19) therefore permits us to gauge away one half on the fermion degrees of freedom. As described below, this gives rise to a matching of physical boson and fermion degrees of freedom on the worldvolume.

3) In the case of the eleven-dimensional supermembrane, it has been shown [16] that the constraints on the background fields  $E_M{}^A$  and  $B_{MNP}$  are nothing but the equations of motion of eleven-dimensional supergravity [49, 70].

# **3.2** Type *IIA* superstring in D = 10 from supermembrane in D = 11

We begin with the bosonic sector of the d = 3 worldvolume of the D = 11 supermembrane:

$$S_{3} = T_{3} \int d^{3}\xi \left[ -\frac{1}{2} \sqrt{-\gamma} \gamma^{ij} \partial_{i} X^{M} \partial_{j} X^{N} G_{MN}(X) + \frac{1}{2} \sqrt{-\gamma} + \frac{1}{3!} \epsilon^{ijk} \partial_{i} X^{M} \partial_{j} X^{N} \partial_{k} X^{P} A_{MNP}(X) \right], \qquad (3.10)$$

where  $T_3$  is the membrane tension,  $\xi^i$  (i = 1, 2, 3) are the worldvolume coordinates,  $\gamma^{ij}$ is the worldvolume metric and  $X^M(\xi)$  are the spacetime coordinates (M = 0, 1, ..., 10). Kappa symmetry [49, 70] then demands that the background metric  $G_{MN}$  and background 3-form potential  $A_{MNP}$  obey the classical field equations of D = 11 supergravity, whose bosonic action is

$$I_{11} = \frac{1}{2\kappa_{11}^2} \int d^{11}x \sqrt{-G} \left[ R_G - \frac{1}{2 \cdot 4!} F_{MNPQ}^2 \right] - \frac{1}{12\kappa_{11}^2} \int A_3 \wedge F_4 \wedge F_4 , \qquad (3.11)$$

where  $F_4 = dA_3$  is the 4-form field strength. In particular,  $F_4$  obeys the field equation

$$d * F_4 = -\frac{1}{2}F_4^2 \tag{3.12}$$

and the Bianchi identity

$$dF_4 = 0 . (3.13)$$

To see how a double worldvolume/spacetime compactification of the D = 11 supermembrane theory on  $S^1$  leads to the Type IIA string in D = 10 [50], let us denote all (d = 3, D = 11) quantities by a hat and all (d = 2, D = 10) quantities without. We then make a ten-one split of the spacetime coordinates

$$\hat{X}^{\hat{M}} = (X^{M}, Y) \qquad M = 0, 1, \dots, 9$$
(3.14)

and a two-one split of the worldvolume coordinates

$$\hat{\xi}^i = (\xi^i, \rho) \qquad i = 1, 2$$
 (3.15)

in order to make the partial gauge choice

$$\rho = Y , \qquad (3.16)$$

which identifies the eleventh dimension of spacetime with the third dimension of the worldvolume. In other words, the membrane is wrapped around the  $S^1$  (See [209] for subtleties concerning zero modes). The dimensional reduction is then effected by taking the background fields  $\hat{G}_{\hat{M}\hat{N}}$  and  $\hat{A}_{\hat{M}\hat{N}\hat{P}}$  to be independent of Y. The string backgrounds of dilaton  $\Phi$ , string  $\sigma$ -model metric  $G_{MN}$ , 1-form  $A_M$ , 2-form  $B_{MN}$  and 3-form  $A_{MNP}$  are given by<sup>10</sup>

$$\hat{G}_{MN} = e^{-\Phi/3} \begin{pmatrix} G_{MN} + e^{\Phi} A_M A_N & e^{\Phi} A_M \\ e^{\Phi} A_N & e^{\Phi} \end{pmatrix}$$
$$\hat{A}_{MNP} = A_{MNP}$$
$$\hat{A}_{MNY} = B_{MN} .$$
(3.17)

The actions (3.10) and (3.11) now reduce to

$$S_{2} = T_{2} \int d^{2}\xi \left[ -\frac{1}{2} \sqrt{-\gamma} \gamma^{ij} \partial_{i} X^{M} \partial_{j} X^{N} G_{MN}(X) -\frac{1}{2!} \epsilon^{ij} \partial_{i} X^{M} \partial_{j} X^{N} B_{MN}(X) + \cdots \right]$$
(3.18)

and

$$I_{10} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} e^{-\Phi} \Big[ R_G + (\partial_M \Phi)^2 - \frac{1}{2 \cdot 3!} H^2_{MNP} - \frac{1}{2 \cdot 2!} e^{\Phi} F^2_{MN} - \frac{1}{2 \cdot 4!} e^{\Phi} J^2_{MNPQ} \Big] - \frac{1}{2\kappa_{10}^2} \int \frac{1}{2} F_4 \wedge F_4 \wedge B_2 , \qquad (3.19)$$

<sup>&</sup>lt;sup>10</sup>The choice of dilaton prefactor,  $e^{-\Phi/3}$ , is dictated by the requirement that  $G_{MN}$  be the D = 10 string  $\sigma$ -model metric. To obtain the D = 10 fivebrane  $\sigma$ -model metric, the prefactor is unity because the reduction is then spacetime only and not simultaneous worldvolume/spacetime. This explains the remarkable "coincidence" [134] between  $\hat{G}_{MN}$  and the D = 10 fivebrane  $\sigma$ -model metric.

where the field strengths are given by  $J_4 = F_4 + A_1H_3$ ,  $H_3 = dB_2$  and  $F_2 = dA_1$ .

One may repeat the procedure in superspace to obtain

$$S_2 = T_2 \int d^2 \xi \left[ -\frac{1}{2} \sqrt{-\gamma} \gamma^{ij} E_i{}^a E_j{}^b \eta_{ab} + \frac{1}{2!} \epsilon^{ij} \partial_i X^M \partial_j X^N B_{MN}(Z) \right]$$
(3.20)

which is just the action of the Type *IIA* superstring.

### 3.3 Bose-fermi matching on the worldvolume

The matching of physical bose and fermi degrees of freedom on the *worldvolume* may, at first sight, seem puzzling since we began with only spacetime supersymmetry. The explanation is as follows. As the *p*-brane moves through spacetime, its trajectory is described by the functions  $X^M(\xi)$  where  $X^M$  are the spacetime coordinates (M = 0, 1, ..., D-1) and  $\xi^i$  are the worldvolume coordinates (i = 0, 1, ..., d-1). It is often convenient to make the so-called *static gauge choice* by making the D = d + (D - d)split

$$X^{M}(\xi) = (X^{\mu}(\xi), Y^{m}(\xi)), \qquad (3.21)$$

where  $\mu = 0, 1, \dots, d-1$  and  $m = d, \dots, D-1$ , and then setting

$$X^{\mu}(\xi) = \xi^{\mu}.$$
 (3.22)

Thus the only physical worldvolume degrees of freedom are given by the  $(D-d) Y^m(\xi)$ . So the number of on-shell bosonic degrees of freedom is

$$N_B = D - d. \tag{3.23}$$

To describe the super *p*-brane we augment the *D* bosonic coordinates  $X^M(\xi)$  with anticommuting fermionic coordinates  $\theta^{\alpha}(\xi)$ . Depending on *D*, this spinor could be Dirac, Weyl, Majorana or Majorana-Weyl. The fermionic kappa symmetry means that half of the spinor degrees of freedom are redundant and may be eliminated by a physical gauge choice. The net result is that the theory exhibits a *d*-dimensional worldvolume supersymmetry [58] where the number of fermionic generators is exactly half of the generators in the original spacetime supersymmetry. This partial breaking of supersymmetry is a key idea. Let *M* be the number of real components of the minimal spinor and *N* the number of supersymmetries in *D* spacetime dimensions and let *m* and *n* be the corresponding quantities in *d* worldvolume dimensions. Let us first consider d > 2. Since kappa symmetry always halves the number of fermionic degrees of freedom and going on-shell halves it again, the number of on-shell fermionic degrees of freedom is

$$N_F = \frac{1}{2}mn = \frac{1}{4}MN.$$
 (3.24)

Worldvolume supersymmetry demands  $N_B = N_F$  and hence

$$D - d = \frac{1}{2}mn = \frac{1}{4}MN.$$
 (3.25)

A list of dimensions, number of real dimensions of the minimal spinor and possible supersymmetries is given in Table 2, from which we see that there are only 8 solutions of (3.25) all with N = 1, as shown in Table 1. We note in particular that  $D_{\text{max}} = 11$ since  $M \ge 64$  for  $D \ge 12$  and hence (3.25) cannot be satisfied. Similarly  $d_{\text{max}} = 6$ since  $m \ge 16$  for  $d \ge 7$ . The case d = 2 is special because of the ability to treat left and right moving modes independently. If we require the sum of both left and right moving bosons and fermions to be equal, then we again find the condition (3.25). This provides a further 4 solutions all with N = 2, corresponding to Type II superstrings in D = 3, 4, 6 and 10 (or 8 solutions in all if we treat Type IIA and Type IIB separately). Both the gauge-fixed Type IIA and Type IIB superstrings will display (8,8) supersymmetry on the worldsheet. If we require only left (or right) matching, then (3.25) replaced by

$$D - 2 = n = \frac{1}{2}MN, (3.26)$$

which allows another 4 solutions in D = 3, 4, 6 and 10, all with N = 1. The gaugefixed theory will display (8,0) worldsheet supersymmetry. The heterotic string falls into this category. The results [58] are indicated by the points labelled s in Table 1. Point particles with d = 1 are usually omitted from the brane-scan [58, 124, 150], but in Table 1 we have included them.

An equivalent way to arrive at the above conclusions is to list all scalar supermultiplets and to interpret the dimension of the target space, D, by

$$D - d =$$
 number of scalars. (3.27)

Indeed, these scalars are the Goldstone bosons associated with the spontaneous breaking of the D-d translations. A useful reference is [48] which provides an exhaustive classification of all unitary representations of supersymmetry with maximum spin 2. In particular, we can understand  $d_{\text{max}} = 6$  from this point of view since this is the upper limit for scalar supermultiplets.

Dimension	Minimal Spinor	Supersymmetry
(D  or  d)	(M  or  m)	(N  or  n)
11	32	1
10	16	2, 1
9	16	2, 1
8	16	2, 1
7	16	2, 1
6	8	4, 3, 2, 1
5	8	4,  3,  2,  1
4	4	8,, 1
3	2	16,, 1
2	1	32,, 1

Table 2: Minimal spinor components and supersymmetries.

There are four types of solution with 8 + 8, 4 + 4, 2 + 2 or 1 + 1 degrees of freedom respectively. Since the numbers 1, 2, 4 and 8 are also the dimension of the four division algebras, these four types of solution are referred to as real, complex, quaternion and octonion respectively. The connection with the division algebras can in fact be made more precise [26, 149, 310, 317, 334].

## **3.4** A heterotic 5-brane?

Of particular interest was the D = 10 fivebrane, whose Wess-Zumino term coupled to a rank six antisymmetric tensor potential  $A_{MNPQRS}$  just as the Wess-Zumino term of the string coupled to a rank two potential  $B_{MN}$ . Spacetime supersymmetry therefore demanded that the fivebrane coupled to the 7-form field strength formulation of D = 10supergravity [17] just as the string coupled to the 3-form version [20, 22]. These dual formulations of D = 10 supergravity have long been something of an enigma from the point of view of superstrings. As field theories, each seems equally valid. In particular, provided we couple them to  $E_8 \times E_8$  or SO(32) super-Yang-Mills [27], then both are anomaly free [47]. Since the 3-form version corresponds to the field theory limit of the heterotic string, we conjectured [54] that there ought to exist a *heterotic fivebrane* which could be viewed as a fundamental anomaly-free theory in its own right and whose field theory limit corresponds to the dual 7-form version. We shall refer to this as the *string/fivebrane duality conjecture.* At this stage, however, the solitonic element had not yet been introduced.

# 3.5 $E(8) \ge SO(16)$ in D=11?

It is interesting to note that the three-eight split

$$SO(1,10) \supset SO(1,2) \times SO(8) \tag{3.28}$$

implied by the embedding of the three-dimensional worldvolume of the supermembrane in eleven-dimensional space-time had previously been invoked in [31] to exhibit the hidden SO(16) symmetry of D = 11 supergravity, where the 128 bosonic degrees of freedom may be assigned to the coset  $E_8/SO(16)$ . We wondered what role  $E_8$ , the Kac-Moody extension  $E_9$  and the Lorentzian algebra  $E_{10}$  will play for the supermembrane.

### 3.6 Branes on the boundary of AdS

Compactification of D=11 supergravity: d = 4 anti-de Sitter space-time x  $S^7$  yields four-dimensional supergravity with maximum (N=8) supersymmetry and local SO(8) invariance [43]. The vacuum symmetry is the AdS supergroup OSp(4/8) which admits the strange "singleton" which have no analogue in the Poincare group and no immediate field theory interpretation. Owing to the N = 8 supersymmetry they form an ultrashort N = 8 supermultiplet consisting of eight spin-1/2 fermions and eight spin-0 bosons which transform according to the 8<sub>s</sub> and 8<sub>v</sub> representations of SO(8). Although we are dealing with the four-dimensional anti de Sitter group SO(2,3), we cannot write down an action for these singletons living in  $AdS_4$ . However, as discussed by Fronsdal [19], we can write down an action living on its three-dimensional boundary  $S^1 \times S^2$  with signature (-, +, +).

But  $8_s$  spin-1/2 and  $8_v$  spin-0 on a 3-dimensional worldvolume with signature (-, +, +) is just what we get from gauge-fixing the supermembrane! We noted that relativistic membranes and singletons have one more thing in common: they were both invented by Dirac at about the same time [1, 2].

### 3.7 Subsequent developments

• Role of D=11 supergravity

Responding to my remark that D = 11 supergravity hints at something beyond strings, Dean Rickles [349] finds it necessary to belittle the role of supergravity compared with superstrings in the historical development of M-theory, calling the years between the discovery of supergravity and the superstring revolution the Decade of Darkness. While it is true that eleven-dimensional quantum supergravity suffers from the ultraviolet divergences that ten-dimensional superstrings avoid, its very existence calls into question the notion that strings are the beall-and-end-all of the final theory. In his zeal to downgrade supergravity Rickles distorts the compliment to make it sound more like an insult: "This became widely accepted, and one can find Michael Duff writing in 1988 that Many of the supergravity theories that we used to study a few years ago are now known to be merely the field theory limit of an underlying string theory."

• M2 brane solutions of D=11 supergravity

The eleven-dimensional supermembrane was subsequently seen to be a solution of the D = 11 supergravity field equations [111] and now plays a vital role in M-theory where it is known as the M2-brane.

• Type IIA string in D=10 from supermembrane in D=11

Witten [154, 163] realises that the radius  $R_{11}$  of the  $S^1$  leads to the Type IIA string with coupling constant  $g_s$  given by

$$g_s = R_{11}^{3/2} \tag{3.29}$$

and we recover the weak coupling regime when  $R_{11} \rightarrow 0$ , which explains the earlier illusion that the theory is defined in D = 10.

• D-branes

Note that if Type II p-branes exist for p > 1, they cannot be described by scalar multiplets [58]. In fact they are described by the vector multiplets that appear on the brane-scan. They subsequently acquired an interpretation as Dirichlet branes, or D-branes [168], surfaces of dimension p on which open strings can end.

• Fivebrane as a soliton

The heterotic 5-brane was found by Strominger to be a soliton solution of the heterotic string [98].

• Exceptional geometry

Exceptional symmetries  $E_8$ ,  $E_9$ ,  $E_{10}$ , appearing not merely upon compactification but already in eleven dimensions, are now the subject of much investigation on the context of exceptional geometry.  $E_{11}$  has taken this one stage further. See section 5.

• AdS/CFT correspondence

Branes on the boundary of AdS are a vital ingredient in the AdS/CFT correspondence [227, 230, 231]. Another vital ingredient, missing in these early days, was the non-abelian nature of the symmetries that appear when we stack N branes on top of one another [170].



Figure 5: From super spaghetti to super ravioli

# 4 1988 Classical and Quantum Supermembranes

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# 4.1 Conformal brane-scan: predicts D3 and M5 in addition to M2

In 1987 two versions of the brane-scan of *D*-dimensional super *p*-branes were put forward. The first by Achucarro, Evans, Townsend and Wiltshire [58] pinpointed those twelve (p, D) slots consistent with kappa-symmetric Green-Schwarz [27] type actions



Julian Schwinger celebrating his 70th Birthday in Erice during the 26th Subnuclear Physics School. From left: Sheldon Glashow, Mrs Manci Dirac, Sergio Ferrara, Michael Duff (1988).

### Figure 6: In distinguished company

for  $p \ge 1$ . The results are the slots labelled s shown in Table 1. Moving diagonally down the brane-scan corresponds to a simultaneous dimensional reduction of spacetime and worldvolume [54]. Of course some of these D dimensions could be compactified, in which case the double dimensional reduction may be interpreted as wrapping the brane around the compactified directions. In each case the boundary CFT is described by the corresponding singleton (scalar), doubleton (scalar or vector) or tripleton (scalar or tensor) supermultiplet. The supersingleton lagrangian and transformation rules were also spelled out explicitly in this paper. Interesting special cases of the conformal brane-scan of Table 3 are (p = 2, D = 11), (p = 3, D = 10) and (p = 5, D = 11) which we now recognise as the M2, D3 and M5 branes. Although the M2 was known, this was the first appearance of D3 and M5. See Table 4. As Steven Weinberg once remarked., the problem with theoretical physicists is not that they take themselves too seriously but that they don't take themselves seriously enough.

Supergroup	AdS Dimension	p
$OSp(p 2) \times OSp(q 2)$	3	1
OSp(N 4)	4	2
SU(2,2 N)	5	3
F(4)	6	4
OSp(6,2 N)	7	5

Table 3: Supergroups admitting *p*-branes on the boundary of  $AdS_{p+2} \times S^{D-p-2}$ .

Supergroup	Spacetime	Brane
OSp(8 4)	$AdS_4 \times S^7$	M2
SU(2, 2 4)	$AdS_5 \times S^5$	D3
OSp(6, 2 4)	$AdS_7 \times S^4$	M5

Table 4: The conformal brane-scan predicts D3 and M5 in addition to M2  $\,$ 

### 4.2 Supermembranes and the signature of spacetime

If our senses are to be trusted, we live in a world with three space and one time dimensions. However, the revival of the Kaluza-Klein idea [143], brought about by supergravity, superstrings and M-theory, has warned us that this may be only an illusion. In any case, there is a hope, so far unfulfilled, that the four-dimensional structure that we apparently observe may actually be predicted by a *Theory of Everything*. Whatever the outcome, imagining a world with an arbitrary number of space dimensions has certainly taught us a good deal about the properties of our three-space-dimensional world.

In spite of all this activity, and in spite of the popularity of Euclidean formulations of field theory, relatively little effort has been devoted to imagining a work with more than one *time* dimension. This is no doubt due partially to the psychological difficulties we have in treating space and time on the same footing. As H. G. Wells reminds us in *The Time Machine*: "There is, however, a tendency to draw an unreal distinction between the former three dimensions and the latter, because it happens that our consciousness moves intermittently in one direction along the latter from the beginning to the end of our lives." There are also more justifiable reasons associated with causality. Nevertheless, one might hope that a theory of everything should predict not only the *dimensionality* of spacetime, but also its *signature*.

For example, quantum consistency of the superstring requires 10 spacetime dimensions, but not necessarily the usual (9,1) signature. The signature is not completely arbitrary, however, since spacetime supersymmetry allows only (9,1), (5,5) or (1,9). Unfortunately, superstrings have as yet no answer to the question of why our universe appears to be four-dimensional, let alone why it appears to have signature (3,1).

In this 1989 lecture therefore I considered a world with an arbitrary number T of time dimensions and an arbitrary number S of space dimensions to see how far classical supermembranes restrict not only S + T but S and T separately. To this end I also allowed an (s,t) signature for the worldvolume of the membrane where  $s \leq S$  and  $t \leq T$  but are otherwise arbitrary. It is not difficult to show that there is once again a matching of the bosonic and fermionic degrees of freedom as a consequence of the kappa symmetry. However severe constraints on possible supermembrane theories will now follow by demanding spacetime supersymmetry [68]. The results are summarized by the *brane-molecule* of Table 5.

Several comments are now in order:

$S\uparrow$													
11													
10		0	$\tilde{O}$										
9	H	H/O	O										
8	$\tilde{C}$	H											
7	$\tilde{C}$	H											
6	$\tilde{C}/\tilde{H}$	H				0	$\tilde{O}$						
5	C	C/H	H	H	H	H/O	0						
4	$\tilde{C}$	C	$\tilde{C}$	$\tilde{C}$	$\tilde{C}$	H							
3		R/C	$\tilde{R}$	$\tilde{R}$	$\tilde{C}$	H							
2		R		$\tilde{R}$	$\tilde{C}$	H				0	$\tilde{O}$		
1			R	R/C	C	C/H	H	H	H	H/O	0		
0					$\tilde{C}$	C	$\tilde{C}/\tilde{H}$	$\tilde{C}$	$\tilde{C}$	H			
	0	1	2	3	4	5	6	7	8	9	10	11	T -

Table 5: The brane molecule

1) We see from Table 5 that for every supermembrane with (S, T) signature, there is another with (T, S). Note the self-conjugate theories that lie on the S = T line which passes through the (5, 5) superstring.

2) There is, as yet, no restriction on the worldvolume signatures beyond the original requirement that  $s \leq S$  and  $t \leq T$ .

3) If we were to redraw the D/d brane-scan of Table 1 allowing now arbitrary signature, there would be no new s points on the plot, but rather the new solutions would be superimposed on the old ones. For example, there would now be six solutions occupying the (d = 3, D = 11) slot instead of one.

4) Perhaps the most interesting aspect of the brane-molecule is the mod 8 periodicity. Suppose there exist signatures (s, t) and (S, T) which satisfy both the requirements of bose-fermi matching and super-Poincare invariance. Now consider (s', t') and (S', T') for which

$$s' + t' = s + t \qquad S' + T' = S + T \tag{4.1}$$

As a consequence of the modulo 8 periodicity theorem for real Clifford algebras, the minimal condition on a spinor is modulo 8 periodic e.g.  $S - T = 0 \mod 8$  for Majorana-Weyl. So if, in addition we also have

$$S' - T' = S - T + 8n \quad n\epsilon Z \quad . \tag{4.2}$$

then (s', t') and (S', T') satisfy bose-fermi matching. (1.52) and (1.53) imply

$$S' = S + 4n \tag{4.3}$$

$$T' = T - 4n \quad . \tag{4.4}$$

Similarly, the membrane at the end of the universe admits a corresponding generalization to brane worldvolumes with s space and t time dimensions moving in a spacetime with  $S \ge s$  space and  $T \ge t$  time dimensions. The brane occupies the boundary of a universe of constant curvature so that the bosonic symmetry is  $O(s + 1, t + 1) \times O(S - s, T - t)$ . Supersymmetry restricts the values of s, t, S, Tto those for which this bosonic symmetry is a subgroup of a superconformal group, and the resulting superconformal theories have  $(s + t) \le 6$ . For example, the possible signatures of M-theory are (10, 1), (9, 2), (6, 5), (5, 6), (2, 9), (1, 10) and the possible M2-branes have worldvolume signatures (3, 0), (2, 1), (1, 2), (0, 3).

### 4.3 D=12?

It is interesting to ask whether we have exhausted all possible theories of extended objects with Green-Schwarz type actions. We demanded super-Poincare invariance but might there exist others for which the supergroup is not necessarily super-Poincare? Although the possibilities are richer, there are still severe constraints. Note, in particular, that the maximum spacetime dimension is now D=12 provided we have signatures (10,2), (6,6) or (2,10). These new cases are particularly interesting since they admit Majorana-Weyl spinors. In fact, twelve-dimensional supersymmetry algebras have been discussed before in the supergravity literature [18]. The RHS of the Q, Q anticommutator yields not only a Lorentz generator but also a six index object so it is certainly not super-Poincare. We conjectured (together with C. Hull and K. Stelle) that the (2, 2) extended object moving in (10,2) spacetime may (if it exists) be related by simultaneous dimensional reduction [50] to the (1,1) Type IIB superstring in (9,1).

### 4.4 Area-preserving diffeomorphisms: Matrix models

In string theory, the light cone gauge is convenient for quantization because it allows the elimination of all unphysical degrees of freedom and unitarity is guaranteed. Of course, one loses manifest Lorentz invariance and one must be careful to check that it is not destroyed by quantization. In membrane theory, however, the lightcone gauge does not eliminate all unphysical degrees of freedom. Let us split

$$X^{\mu} = (X^{\pm}, X^{I}) \qquad I = 1, 2, ... (D - 2)$$
$$X^{\pm} = \frac{1}{\sqrt{2}} (X^{0} \pm X^{D-1})$$
(4.5)

One can then solve for  $X^-$  leaving the (D-2) variables  $X^I$ . For membranes, however, only (D-d) variables are physical. Thus the light-cone gauge must leave a residual gauge invariance [21, 45]. This group is, in fact, the subgroup of the worldvolume diffeomorphism group that preserves the Lie bracket

$$\{f,g\} = \epsilon^{ab} \partial_a f \partial_b g \tag{4.6}$$

and is known as the group of area-preserving diffeomorphisms. For spherical membranes, this group is given by  $\lim N \to \infty SU(N)$ . Let us focus our attention on a d = 3 supermembrane in flat spacetime. The light-cone action turns out to be

$$S = \frac{1}{2} \int d\tau tr\{(D_0 A^I)^2 - \frac{1}{2} [A^I, A^J] [A^I, A^J] + i\bar{\lambda}D_0\lambda + i\bar{\lambda}\gamma^I [A^I, \lambda]\}$$
(4.7)

where the fields are all in the adjoint representation of  $SU(\infty)$ . Remarkably, this looks like a (D-1)-dimensional super-Yang-Mills theory dimensionally reduced to one time dimension.

One can generalise these results to the d = 3 supermembranes [73] in D = 4, 5, 7and 11, and one finds super-Yang-Mills quantum mechanical models corresponding to the dimensional reduction of super-Yang-Mills in D = 3, 4, 6 and 10, which provides yet another way of understanding the allowed values of D. (One might conjecture a similar relationship between the d > 3 membranes and quantum mechanical models, but this time the gauge symmetry could not be of the Yang-Mills type. It has been suggested [49] that they are given by infinite-dimensional non-Abelian antisymmetric tensor gauge theories.)

## 4.5 Subsequent developments

### • Black branes

Following the *electric* M2-brane solution of D = 11 supergravity [111], the dual *magnetic* M5-brane solution was found by Gueven [118]. The black p-brane solution of Types IIA and IIB supergravity were found by Horowitz and Strominger

[107] and the extremal cases were proven to be supersymmetric (1/2 BPS) in [112, 124]. In his now famous paper, Polchinski [168] provided an alternative derivation as Dirichlet-branes on which open strings may end.

• Branes and the signature of spacetime

Branes in exotic signatures were further studied by Hull [237], Hull and Khuri [256], Batrachenko, Duff and Lu [267], Duff and Kalakinin [289, 290]. Note. however, that, signature reversal  $(S,T) \rightarrow (T,S)$  in general yields a different theory. The conditions for reversal invariance for both supergravities and branes are spelled out in [289, 290]. A necessary but not sufficient conditon is that the Clifford algebra obey Cliff(S,T)=Cliff(T,S) which requires  $S - T = 0 \mod 4$ . Physics with more than one time has also been pursued by Bars [304]. Negative branes, supergroups, and the signature of spacetime was the subject of a recent paper by Dijkgraaf, Heidenreich, Jefferson and Vafa [357]. A supergravity lagrangian in (10, 2) was recently proposed by Castellani [366].

• F-theory

The idea of a 12-dimensional world was revived by Vafa in the context of Ftheory [187], which involves Type IIB compactification where the axion from the R-Rsector and dilaton from the NS-NS sector are allowed to vary on the internal manifold. Given a manifold M that has the structure of a fiber bundle whose fiber is  $T^2$  and whose base is some manifold B, then

$$F \text{ on } M \equiv Type IIB \text{ on } B \tag{4.8}$$

### • Matrix models

The  $SU(\infty)$  Yang-Mills description of M2-branes was revived by Banks, Fischler, Shenker and Susskind in the matrix-model interpretation of M-theory [210], which has received some recent attention by Maldacena and Milekhin [368].

• Holographic duals

Although the D3 worldvolume theory on the boundary of  $AdS_5 \times S^5$  is the wellknown N = 4 Yang-Mills, the holographic duals of M-theory on  $AdS_4 \times S^7$  and  $AdS_7 \times S^4$  are more obscure. ABJM (Aharony, Bergman, Jafferis, Maldacena) theory [305] is the favorite candidate for M2 but the M5 case is a (2,0) CFT not describable by a lagrangian field theory. See also [299]. In this context and in the context of exotic signatures, it is worth bearing in mind that the (9,2) version of M-theory admits a doubly holographic  $AdS_4 \times AdS_7$  solution. This may be regarded either as a stack of M2 branes with conformal group SO(4,2) and R-symmetry SO(6,2), or as a stack of M5 branes for which the conformal and R-symmetries are interchanged [255, 256, 267]. Perhaps the ABJM of one can throw light on the (2,0) CFT of the other.

# 5 1990 Symmetries of Extended Objects

INTERNATIONAL SCHOOL OF SUBNUCLEAR PHYSICS - Director: A. ZICHICHI 28th Course: Physics up to 200 TeV 16 - 24 July 1990 [102]

# 5.1 T-duality and double geometry: $Z^M = (x^{\mu}, y_{\alpha})$

In this 1990 lecture, based on an earlier paper [93], I pointed out that strings moving in an *n*-dimensional space  $M^n$  with coordinates  $X^{\mu}(\tau, \sigma)$ , background metric  $g_{\mu\nu}(X)$  and 2-form  $b_{\mu\nu}(X)$ , could usefully be described by a doubled geometry with 2*n*-dimensional coordinates

$$Z^M = (X^\mu, Y_\sigma) \tag{5.1}$$

and doubled metric<sup>11</sup>

$$G_{MN} = \begin{pmatrix} g_{\mu\nu} - b_{\mu\rho} g^{\rho\sigma} b_{\sigma\nu} & b_{\mu\rho} g^{\rho\sigma} \\ -g^{\mu\sigma} b_{\sigma\nu} & g^{\mu\nu} \end{pmatrix}$$
(5.2)

The motivation was twofold; worldsheet and spacetime:

### 1. Worldsheet

In the case when  $M^n$  is the *n*-torus  $T^n$ , this renders manifest the O(n, n) Tduality by combining worldsheet field equations and Bianchi identities via the constraint

$$\Omega_{MN}\epsilon^{ij}\partial_j Z^N = G_{MN}\sqrt{-\gamma}\gamma^{ij}\partial_j Z^N \,, \tag{5.3}$$

where

$$\Omega_{MN} = \begin{pmatrix} 0 & \delta_{\mu}{}^{\beta} \\ \delta^{\alpha}{}_{\nu} & 0 \end{pmatrix}, \qquad (5.4)$$

and  $\gamma_{ij}$  is the worldsheet metric. In components

$$\epsilon^{ij}\partial_j Y_{\nu} = \sqrt{-\gamma}\gamma^{ij}\partial_j X^{\mu}g_{\mu\nu} + \epsilon^{ij}\partial_j X^{\mu}b_{\mu\nu}$$

 $<sup>^{11}</sup>G_{MN}$  had previously appeared in [92] with a different physical interpretation as a metric on phase space.

$$\epsilon^{ij}\partial_j X^{\nu} = \sqrt{-\gamma}\gamma^{ij}\partial_j Y_{\mu}p^{\mu\nu} + \epsilon^{ij}\partial_j Y_{\nu}q^{\mu\nu}$$

Here  $Y_{\nu}(\tau, \sigma)$  are the coordinates of the T-dual string which interchanges field equations and bianchi identities. Its background metric and 2-form are  $p^{\mu\nu}(X)$ and  $q^{\mu\nu}(X)$  where

$$g = p^{-1}(1 - qb) \quad b = -p^{-1}qg$$
 (5.5)

$$p^{-1} = g - bg^{-1}b \quad g^{-1}b = -pq^{-1} \tag{5.6}$$

 $\mathbf{SO}$ 

$$(p \pm q)(g \pm b) = 1$$
 (5.7)

An earlier alternative suggestion [42] was to use the non-symmetric metric  $g_{\mu\nu} + b_{\mu\nu}$ . The two alternatives are related by the two-vielbein (left L and right R) approach [129].

$$\begin{pmatrix} e_{\mu}{}^{a}(L)e_{\nu a}(R) & e_{\mu}{}^{a}(L)e^{\nu}{}_{a}(R) \\ e^{\mu a}(L)e_{\nu a}(R) & e^{\mu a}(L)e^{\nu}{}_{a}(R) \end{pmatrix} = \begin{pmatrix} g_{\mu\nu} + b_{\mu\nu} & \delta_{\mu}{}^{\nu} \\ \delta^{\mu}{}_{\nu} & p^{\mu\nu} + q^{\mu\nu} \end{pmatrix}$$
(5.8)

#### 2. Spacetime

In the case when  $M^n$  is a generic manifold, the 2*n*-dimensional diffeomorphisms with parameter  $\xi^M = (\xi^{\mu}, \lambda_{\alpha})$  suggest a way of unifying *n*-dimensional diffeomorphisms

$$\delta g_{\mu\nu} = -\partial_{\mu}\xi^{\rho}g_{\rho\nu} - \partial_{\nu}\xi^{\rho}g_{\mu\rho} - \partial_{\rho}g_{\mu\nu}\xi^{\rho}, \qquad (5.9)$$

and 2-form gauge invariance

$$\delta B_{\mu\nu} = \partial_{\mu}\lambda_{\nu} - \partial_{\nu}\lambda_{\mu} \,. \tag{5.10}$$

After all,  $G_{MN}$  is just the Kaluza-Klein metric with spacetime metric  $g_{\mu\nu}$ , gauge field  $A_{\mu}{}^{a}$  and internal metric  $g_{ab}$ 

$$G_{MN} = \begin{pmatrix} g_{\mu\nu} + A_{\mu}{}^{a}g_{ab}A_{\nu}{}^{b} & A_{\mu}{}^{a}g_{ab} \\ g_{ab}A_{\nu}{}^{b} & g_{ab} \end{pmatrix},$$
(5.11)

where the "gauge field" is  $B_{\mu\alpha}$  and the "internal" metric is  $g^{\alpha\beta}$ . If this programme were successful one would expect the SL(n)/SO(n) coset of general relativity to be promoted to an  $O(n,n)/(SO(n) \times SO(n))$ , as conjectured in [31, 42].

n	G	Н	dim G/H	$n(n^2+5)/3!$	_
1	R	1	1	1	√
2	GL(2, ℝ)	SO(2)	3	3	$\checkmark$
3	$SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$	$SO(3) \times SO(2)$	7	7	$\checkmark$
4	SL(5, ℝ)	SO(5)	14	14	$\checkmark$
5	SO(5,5)	$SO(5) \times SO(5)$	25	25	V
6	$E_{6(+6)}$	USp(8)	42	41	×
7	$E_{7(+7)}$	SU(8)	70	63	×
8	$E_{8(+8)}$	SO(16)	128	92	×

The hidden global symmetries G and local symmetries H that result from compactifying D = 11supergravity on T<sup>n</sup>. For  $n \le 5$ , these are compatible with the coset parametrized by the membrane background fields  $g_{\mu\nu}$  and  $b_{\mu\nu\rho}$  only. Extra space-time scalars must be included for  $n \ge 6$ 

### Figure 7: U-dualities and branes

Similarly, based on [99, 100], I pointed out that membranes moving in a  $(n \leq 4)$ dimensional space  $M^n$  with coordinates  $X^{\mu}(\tau, \sigma, \rho)$ , background metric  $g_{\mu\nu}(X)$  and 3-form  $B_{\mu\nu\rho}(X)$  could usefully be described by a geometry with [n + n(n - 1)/2]dimensional coordinates

$$Z^M = (X^\mu, Y_{\rho\sigma}) \tag{5.12}$$

and generalized metric

$$G_{MN} = \begin{pmatrix} g_{\mu\nu} + b_{\mu\rho\sigma} g^{\rho\sigma\lambda\tau} b_{\lambda\tau\nu} & b_{\mu\rho\sigma} g^{\rho\sigma\lambda\tau} \\ g^{\mu\nu\rho\sigma} b_{\rho\sigma\nu} & g^{\mu\nu\rho\sigma} \end{pmatrix}, \qquad (5.13)$$

where

$$g^{\alpha\beta\gamma\delta} = \frac{1}{2} (g^{\alpha\gamma} g^{\beta\delta} - g^{\alpha\delta} g^{\beta\gamma}).$$
 (5.14)

Once again, the motivation was twofold; worldvolume and spacetime:

### 1. Worldvolume

In the case when  $M^n$  is the *n*-torus  $T^n$ , the hope was to render manifest the M-theory U-dualities (using modern parlance) by combining worldvolume field equations and Bianchi identities. For example, the U-duality would be SL(5, R) in the case n = 4. The restriction to  $n \leq 4$  arises because, just as the usual coordinates  $X^{\mu}$  correspond to momentum in the supersymmetry algebra, so the

n	G	spin-1 reps.	n(n+1)/2	
1	R	1	1	√
2	GL(2, ℝ)	3	3	$\checkmark$
3	$SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$	(3, 2)	6	$\checkmark$
4	SL(5, ℝ)	10	10	$\checkmark$
5	SO(5,5)	16	15	×
6	$E_{6(\pm 6)}$	27	21	×
7	$E_{7(+7)}$	56	28	×
8	$E_{8(+8)}$	_	36	×

Representations of the duality symmetry under which the spin-1 space-time fields of compactified D = 11 supergravity transform. For  $n \le 4$ , these are compatible with the representations of the membrane "field strengths"  $\tilde{\mathscr{I}}^i_{\mu}$  and  $\tilde{\mathscr{F}}^{i\mu\nu}$  only. Extra field strengths must be included for  $n \ge 5$ 

#### Figure 8: U-dualities and branes

extra coordinates  $Y_{\mu\nu}$  correspond to the M2 central charge. But for  $n \geq 5$ , this is not enough, as shown in Fig.8. There is also the M5 central charge with corresponding coordinates  $Y_{\mu\nu\rho\sigma\tau}$ , which first appears in n = 5. In general there are extra coordinates for all central charges in the M-theory algebra for general D. For example, in the n = 7 case  $X^{\mu}$ ,  $Y_{\mu\nu}$ ,  $\tilde{Y}^{\mu\nu} \sim \epsilon^{\mu\nu\rho\sigma\tau\lambda\kappa}Y_{\rho\sigma\tau\lambda\kappa}$  and  $\tilde{X}^{\mu}$  form a 56 of the U-duality symmetry  $E_{7(7)}$ .

#### 2. Spacetime

If this programme were successful, one would expect the SL(n)/SO(n) of general relativity to be promoted not merely to  $O(n,n)/(SO(n) \times SO(n))$  but to  $E_8/SO(16)$ , with possible infinite-dimensional extensions involving  $E_9$ ,  $E_{10}$  as conjectured in [31, 42]

## 5.3 Subsequent developments

• Dual variables

The variables p and q re-appear in the literature on noncommutative geometry [249], in non-geometric flux and  $\beta$ -supergravity where they are known as  $\tilde{g}$  and  $\beta$  [326, 323, 324] and in Yang-Baxter equations [370].

• Double field theory

The worldsheet goal of rendering manifest the string T-duality O(n, n) by doubling the coordinates was achieved successfully in [93] and a T-dual worldsheet

action using the doubled coordinates was then constructed in [101]. There were missing ingredients in the spacetime approach: The generalized diffeomorphisms were subsequently supplied in [129, 131]

$$\delta G_{MN} = \xi^P \partial_P G_{MN} + (\partial_M \xi^P - \partial^P \xi_M) G_{PN} + (\partial_N \xi^P - \partial^P \xi_N) G_{MP}, \quad (5.15)$$

and the section condition subsequently supplied in [277]

$$\Omega^{MN} \partial_M \partial_N = 0. (5.16)$$

(The need for the section condition has, however, been called into question [322, 132].) Once these ingredients were included, it was possible also to build a generalised spacetime action for  $G_{MN}$ . This activity came to be known as "Double Field Theory." [277, 308, 311, 313, 315]

• Anachronisms

Although it is quite common for papers on Double Geometry to begin with its history, they often contain anachronisms, in the sense that results of paper A are said to extend those of paper B even though A preceded B. In particular, the chronology of the papers by Duff in August 1989 [93], by Tseytlin in February 1990 [101] and June 1990 [113] and by Siegel in February 1993 [129], May 1993 [131] and August 1993 [136] is frequently reversed. For example in [350] reference [93] is described as a "descendant" of [101], in the first version of [329] as an "elaboration" of [101, 113], in [330] as a "development" of [129, 131] and in [351] as "recent". The fact that [93] is pre-arXiv may be a contributing factor to the ordering ambiguities.

• Exceptional field theory

Similarly with the supermembrane, the generalised diffeomorphisms, section conditions and U-invariant actions came later. These activities involved *Generalized Geometry* [327], *Exceptional Field Theory* [277, 308, 311, 313, 315, 338, 344, 345, 346] and  $E_{11}$  [263]. For subsequent developments and variations on generalized geometry in M-theory and U-duality see, for example, [303, 313, 315, 319, 320, 346] where the 5-brane and other extended objects were incorporated, as required for n > 4. The  $E_{11}$  approach [263] goes further with infinitely many coordinates of which those associated with the M-theory central charges are but a subset. In summary, in contrast with strings where both the worldsheet and spacetime

approaches have been successful, the brane worldvolume approach seems prob-

lematical [142, 181, 222, 353] and, with the exception of [319, 345, 353], recent developments have tended to focus on the spacetime approach where the extra coordinates (5.12) and generalised metric (5.13) have proved valuable. In any event, the need to include coordinates corresponding to central charges in the M-theory algebra exposes a major difference between U-duality in M-theory and T-duality in string theory. In string theory, T-duality takes strings into strings, but in M-theory U-duality mixes up *p*-branes with different *p*. It seems unlikely, therefore, that the M2-brane worldvolume alone is sufficient. Somehow the totality of *p*-brane worldvolumes must conspire to give the full U-duality. This remains an unsolved problem.

# 6 1991 A Duality between Strings and 5-branes

INTERNATIONAL SCHOOL OF SUBNUCLEAR PHYSICS - Director: A. ZICHICHI 29th Course: Physics at the Highest Energy and Luminosity: to Understand the Origin of Mass 14 - 22 July 1991 [364]

## 6.1 Elementary v solitonic branes

The next development came when Townsend [67] pointed out that all the points on the  $\mathcal{H}, \mathcal{C}, \mathcal{R}$  branescan sequences correspond to topological defects of some globally supersymmetric field theory which break half the spacetime supersymmetries. He conjectured that the p-branes in the  $\mathcal{O}$  sequence would also admit such a solitonic interpretation within the context of supergravity. The first hint in this direction came from Dabholkar et al. [97], who presented a multi-string solution which in D = 10indeed breaks half the supersymmetries. They obtained the solution by solving the low-energy 3-form supergravity equations of motion coupled to a string  $\sigma$ -model source and demonstrated that it saturated a Bogomol'nyi bound and satisfied an associated zero-force condition, these properties being intimately connected with the existence of unbroken spacetime supersymmetry. However, this D = 10 string was clearly not the soliton anticipated by Townsend because it described a singular configuration with a  $\delta$ -function source at the string location. Moreover, its charge per unit length  $e_2$  was an "electric" Noether charge associated with the equation of motion of the antisymmetric tensor field rather than a "magnetic" topological charge associated with the Bianchi identities. Consequently, in the current literature on the subject, this solution is now
referred to as the "fundamental" or "elementary" string.

Similarly, the supermembrane solution of D = 11 supergravity found in [111] was not solitonic either because it was also obtained by coupling to a membrane  $\sigma$ -model source. Curiously, however, the curvature computed from its  $\sigma$ -model metric is finite at the location of the source, in contrast to the case of the elementary string.

The next major breakthrough for p-branes as solitons came with the paper of Strominger [98], who showed that D = 10 supergravity coupled to super Yang-Mills (without a  $\sigma$ -model source), which is the field theory limit of the heterotic string [35], admits as a solution the heterotic fivebrane. In contrast to the elementary string, this fivebrane is a genuine soliton, being everywhere nonsingular and carrying a topological magnetic charge  $g_6$ . A crucial part of the construction was a Yang-Mills instanton in the four directions transverse to the fivebrane. He went on to suggest a complete strong/weak coupling duality with the strongly coupled string corresponding to the weakly coupled fivebrane and vice-versa, thus providing a solitonic interpretation of the string/fivebrane duality conjecture. In this form, string/fivebrane duality is in a certain sense an analog of the Montonen-Olive [7] according to which the magnetic monopole states of four-dimensional spontaneously broken supersymmetric Yang-Mills theories may be viewed from a dual perspective as fundamental in their own right and in which the roles of the elementary and solitonic states are interchanged.

This strong/weak coupling theme was further developed in [103] which also established a Dirac quantization rule

$$\kappa^2 T_2 T_6 = n\pi, \qquad n = \text{integer}$$
(6.1)

relating the fivebrane tension  $T_6$  to the string tension  $T_2$ , which followed from the corresponding rule for the electric and magnetic charges generalized to extended objects [33, 40]  $e_2g_6 = 2n\pi$ .

For the purposes of generalizing the Dirac quantization rule for extended objects, we recall that just as a charged particle couples to an Abelian vector potential  $A_M$ displays a gauge invariance

$$A_M \to A_M + \partial_M \Lambda \tag{6.2}$$

and has a gauge invariant field strength

$$F_{MN} = 2\partial_{[M}A_{N]} \equiv \partial_{M}A_{N} - \partial_{N}A_{M}, \tag{6.3}$$

a string couples to a rank-2 antisymmetric tensor potential  $A_{MN} = -A_{NM}$  with a

gauge invariance

$$A_{MN} \to A_{MN} + \partial_{[M} \Lambda_{N]}, \tag{6.4}$$

and field strength

$$F_{MNP} = 3\partial_{[M}A_{NP]}.\tag{6.5}$$

In general, a (d-1)-brane couples to a d-form  $A_{M_1M_2\cdots M_d}$  with

$$A_{M_1M_2\cdots M_d} \to A_{M_1M_2\cdots M_d} + \partial_{[M_1}\Lambda_{M_2\cdots M_d]},\tag{6.6}$$

and

$$F_{M_1M_2\cdots M_{d+1}} = (d+1)\partial_{[M_1}A_{M_2\cdots M_{d+1}]}.$$
(6.7)

In the language of differential forms we may write for arbitrary d and D

$$A_d \to A_d + d\Lambda_{d-1},\tag{6.8}$$

and

$$F_{d+1} = dA_d, \tag{6.9}$$

from which the Bianchi identity

$$dF_{d+1} \equiv 0 \tag{6.10}$$

follows immediately. In the absence of other interactions, the equation of motion for the d-form potential is

$$d^*F_{D-d-1} = {}^*J_{D-d}, (6.11)$$

where the source J is a d-form. Here we have introduced the Hodge dual operation \* which converts a d-form into a (D - d)-form, e.g.

$$({}^{*}J)^{M_{1}M_{2}\cdots M_{D-d}} \equiv \frac{1}{d!} \varepsilon^{M_{1}M_{2}\cdots M_{D}} J_{M_{D-d+1}\cdots M_{D}},$$
 (6.12)

where  $\varepsilon^{M_1 \cdots M_D}$  is the *D*-dimensional alternating symbol with  $\varepsilon^{01 \cdots D-1} = 1$ .

In analogy with the usual Maxwell's equations, (6.11) and (6.10) imply the presence of an "electric" charge, i.e. a (d-1)-brane, but no "magnetic" charge, i.e. no (D-d-3)brane. To restore the duality symmetry by introducing a (D-d-3)-brane we must modify (6.9) to

$$F_{d+1} = dA_d + \omega_{d+1}, (6.13)$$

so that the Bianchi identity becomes

$$dF_{d+1} = X_{d+2}, (6.14)$$

with

$$X_{d+2} = d\omega_{d+1}.$$
 (6.15)

X may be singular

$$X_{123\cdots d+2} = g_{D-d-2}\delta^{d+2}(y), \tag{6.16}$$

or may be smeared out so as to be regular at the origin. We then have

$$e_d = \int_{S^{D-d-1}} {}^*F_{D-d-1} = \int_{M^{D-d}} {}^*J_{D-d}, \tag{6.17}$$

$$g_{D-d-2} = \int_{S^{d+1}} F_{d+1} = \int_{M^{d+2}} X_{d+2}.$$
 (6.18)

The Dirac quantization condition is again obtained by using the generalization of either the Dirac string [40] or Wu-Yang construction [33] as

$$\frac{e_d g_{D-d-2}}{4\pi} = \frac{1}{2}(n = \text{integer})$$
 (6.19)

Note that,  $e_d$  and  $g_{D-d-2}$  are not in general dimensionless but rather

$$[e_d] = -\frac{1}{2}(D - 2d - 2), \qquad [g_{D-2d-2}] = \frac{1}{2}(D - 2d - 2). \tag{6.20}$$

They do become dimensionless when

$$D = 2(d+1), (6.21)$$

of which the point particle (d = 1) in D = 4 is the most familiar special case.

We shall now consider the elementary string where  $X_8$  is singular, the solitonic fivebrane where  $X_4$  is zero, the elementary fivebrane where  $X_4$  is singular and the solitonic string where  $X_8$  is zero. The solitons are regular in the sense that the curvature singularities are absent when written in terms of the corresponding dual frame sigmamodel metrics given below [127]. Moreover a probe fivebrane takes infinite proper time to fall onto a heavy string and vice versa.

Then we allow for the presence of Yang-Mills fields and consider the solitonic string where  $X_8$  is non-zero but regular and the solitonic fivebrane where  $X_4$  is non-zero but regular.

#### 6.2 The elementary string and solitonic fivebrane

We begin by recalling the elementary string solution of [97]. We want to find a vacuumlike supersymmetric configuration with D = 2 super-Poincare symmetry from the 3form version of D = 10, N = 1 supergravity theory. As usual, the fermionic fields should vanish for this configuration. We start by making an ansatz for the D = 10 metric  $g_{MN}$ , 2-form  $B_{MN}$  and dilaton  $\phi$  ( $M = 0, 1, \dots, 9$ ) corresponding to the most general eight-two split invariant under  $P_2 \times SO(8)$ , where  $P_2$  is the D = 2 Poincare group. We split the indices

$$x^{M} = (x^{\mu}, y_{m}), \tag{6.22}$$

where  $\mu = 0, 1$  and  $m = 2, \dots, 9$ , and write the line element as

$$ds^{2} = e^{2A} \eta_{\mu\nu} dx^{\mu} dx^{\nu} + e^{2B} \delta^{mn} dy_{m} dy_{n}, \qquad (6.23)$$

and the two-form gauge field as

$$B_{01} = -e^C. (6.24)$$

All other components of  $B_{MN}$  and all components of the gravitino  $\psi_M$  and dilatino  $\lambda$ are set zero.  $P_2$  invariance requires that the arbitrary functions A, B and C depend only on  $y^m$ ; SO(8) invariance then requires that this dependence be only through  $y = \sqrt{\delta_{mn} y^m y^n}$ . Similarly, our ansatz for the dilaton is

$$\phi = \phi(y). \tag{6.25}$$

As we shall now show, the four arbitrary functions A, B, C, and  $\phi$  are reduced to one by the requirement that the above field configurations preserve some unbroken supersymmetry. In other words, there must exist Killing spinors  $\varepsilon$  satisfying [97]

$$\delta\psi_M = D_M\varepsilon + \frac{1}{96} e^{-\phi/2} (\Gamma_M {}^{NPQ} - 9 \delta_M {}^N \Gamma^{PQ}) H_{NPQ} \varepsilon = 0, \qquad (6.26)$$

$$\delta\lambda = -\frac{1}{2\sqrt{2}} \Gamma^M \partial_M \phi \varepsilon + \frac{1}{24\sqrt{2}} e^{-\phi/2} \Gamma^{MNP} H_{MNP} \varepsilon = 0, \qquad (6.27)$$

where

$$H_{MNP} = 3\partial_{[M}A_{NP]}.$$
(6.28)

Here  $\Gamma_A$  are the D = 10 Dirac matrices satisfying

$$\{\Gamma_A, \Gamma_B\} = 2\eta_{AB}.\tag{6.29}$$

A, B refer to the D = 10 tangent space,  $\eta_{AB} = (-, +, \dots, +)$ , and

$$\Gamma_{AB\cdots C} = \Gamma_{[A}\Gamma_{B\cdots}\Gamma_{C]},\tag{6.30}$$

thus  $\Gamma_{AB} = \frac{1}{2} (\Gamma_A \Gamma_B - \Gamma_B \Gamma_A)$ , etc. The  $\Gamma$ 's with world-indices  $P, Q, R, \cdots$  have been converted using vielbeins  $e_M^A$ . We make an eight-two split

$$\Gamma_A = (\gamma_\alpha \otimes 1, \gamma_3 \otimes \Sigma_a), \tag{6.31}$$

where  $\gamma_{\alpha}$  and  $\Sigma_a$  are the D = 2 and D = 8 Dirac matrices, respectively. We also define

$$\gamma_3 = \gamma_0 \gamma_1, \tag{6.32}$$

so that  $\gamma_3^2 = 1$  and

$$\Gamma_9 = \Sigma_2 \Sigma_3 \cdots \Sigma_9, \tag{6.33}$$

so that  $\Gamma_9^2 = 1$ . The most general spinor consistent with  $P_2 \times SO(8)$  invariance takes the form

$$\varepsilon(x,y) = \epsilon \otimes \eta, \tag{6.34}$$

where  $\epsilon$  is a spinor of SO(1,1) which may be further decomposed into chiral eigenstates via the projection operators  $(1 \pm \gamma_3)$  and  $\eta$  is an SO(8) spinor which may further be decomposed into chiral eigenstates via the projection operators  $(1 \pm \Gamma_9)$ . The N = 1, D = 10 supersymmetry parameter is, however, subject to the ten-dimensional chirality condition

$$\Gamma_{11} \ \varepsilon = \varepsilon, \tag{6.35}$$

where  $\Gamma_{11} = \gamma_3 \otimes \Gamma_9$  and so the D = 2 and D = 8 chiralities are correlated.

Substituting the ansatz into the supersymmetry transformation rules leads to the solution [97]

$$\varepsilon = e^{3\phi/8} \epsilon_0 \otimes \eta_0, \tag{6.36}$$

where  $\epsilon_0$  and  $\eta_0$  are constant spinors satisfying

$$(1 - \gamma_3)\epsilon_0 = 0, \quad (1 - \Gamma_9)\eta_0 = 0,$$
 (6.37)

and where

$$A = \frac{3\phi}{4} + c_A,\tag{6.38}$$

$$B = -\frac{\phi}{4} + c_B,\tag{6.39}$$

$$C = 2\phi + 2c_A,\tag{6.40}$$

where  $c_A$  and  $c_B$  are constants. If we insist that the metric is asymptotically Minkowskian, then

$$c_A = -\frac{3\phi_0}{4}, \quad c_B = \frac{\phi_0}{4},$$
 (6.41)

where  $\phi_0$  is the value of  $\phi$  at infinity *i.e.* the dilaton vev  $\phi_0 = \langle \phi \rangle$ . The condition (6.37) means that one half of the supersymmetries are broken.

At this stage the four unknown functions A, B, C and  $\phi$  have been reduced to one by supersymmetry. To determine  $\phi$ , we must substitute the ansatz into the field equations which follow from the action  $I_{10}(\text{string}) + S_2$  where  $I_{10}(\text{string})$  is the bosonic sector of the 3-form version of D = 10, N = 1 supergravity given by

$$I_{10}(\text{string}) = \frac{1}{2\kappa^2} \int d^{10}x \ \sqrt{-g} \left( R - \frac{1}{2} (\partial \phi)^2 - \frac{1}{2 \cdot 3!} \ e^{-\phi} H^2 \right), \tag{6.42}$$

and  $S_2$  is the string  $\sigma$ -model action. Up until now we have employed the canonical choice of metric for which the gravitational action is the conventional Einstein-Hilbert action. This metric is related to the metric appearing naturally in the string  $\sigma$ -model by

$$g_{MN}(\text{string } \sigma\text{-model}) = e^{\phi/2} g_{MN}(\text{canonical}),$$
 (6.43)

In canonical variables, therefore, the string  $\sigma$ -model action is given by

$$S_{2} = -T_{2} \int d^{2}\xi \left(\frac{1}{2}\sqrt{-\gamma} \gamma^{ij}\partial_{i}X^{M}\partial_{j}X^{N}g_{MN} e^{\phi/2} - 2\sqrt{-\gamma} + \frac{1}{2!} \varepsilon^{ij}\partial_{i}X^{M}\partial_{j}X^{N}B_{MN}\right).$$

$$(6.44)$$

We have denoted the string tension by  $T_2$ . The supergravity field equations are

$$R^{MN} - \frac{1}{2} \left( \partial^{M} \phi \ \partial^{N} \phi - \frac{1}{2} \ g^{MN} (\partial \phi)^{2} \right) - \frac{1}{2} \ g^{MN} R - \frac{1}{2 \cdot 2!} \left( H^{M}_{PQ} H^{NPQ} - \frac{1}{6} \ g^{MN} H^{2} \right) e^{-\phi} = \kappa^{2} T^{MN} (\text{string}), \quad (6.45)$$

where

$$T^{MN}(\text{string}) = -T_2 \int d^2 \xi \sqrt{-\gamma} \ \gamma^{ij} \partial_i X^M \partial_j X^N e^{\phi/2} \ \frac{\delta^{10}(x-X)}{\sqrt{-g}}, \tag{6.46}$$

$$\partial_M(\sqrt{-g} \ e^{-\phi} H^{MNP}) = 2\kappa^2 T_2 \int d^2\xi \ \varepsilon^{ij} \partial_i X^N \partial_j X^P \delta^{10}(x-X), \tag{6.47}$$

$$\partial_M(\sqrt{-g}g^{MN}\partial_N\phi) + \frac{1}{2\cdot 3!} e^{-\phi}H^2 =$$
  
=  $\frac{\kappa^2 T_2}{2} \int d^6\xi \sqrt{-\gamma} \gamma^{ij} \partial_i X^M \partial_j X^N g_{MN} e^{\phi/2} \delta^{10}(x-X).$  (6.48)

Furthermore, the string field equations are

$$\partial_i (\sqrt{-\gamma} \gamma^{ij} \partial_j X^N g_{MN} e^{\phi/2}) - \frac{1}{2} \sqrt{-\gamma} \gamma^{ij} \partial_i X^N \partial_j X^P \partial_M (g_{NP} e^{\phi/2}) - \frac{1}{2} \varepsilon^{ij} \partial_i X^N \partial_j X^P H_{MNP} = 0, \qquad (6.49)$$

and

$$\gamma_{ij} = \partial_i X^M \partial_j X^N g_{MN} e^{\phi/2}. \tag{6.50}$$

To solve these coupled supergravity-string equations we make the static gauge choice

$$X^{\mu} = \xi^{\mu}, \quad \mu = 0, 1 \tag{6.51}$$

and the ansatz

$$X^m = Y^m = \text{constant}, \qquad m = 2, ..., 9.$$
 (6.52)

As an example, let us now substitute the ansatz into and the 2-form equation. We find

$$\delta^{mn}\partial_m\partial_n e^{-2\phi} = -2\kappa^2 T_2 e^{-\phi_0/2} \delta^8(y), \tag{6.53}$$

and hence

$$e^{-2\phi} = e^{-2\phi_0} \left( 1 + \frac{k_2}{y^6} \right), \tag{6.54}$$

where the constant  $k_2$  is given by

$$k_2 \equiv \frac{\kappa^2 T_2}{3\Omega_7} \ e^{3\phi_0/2},\tag{6.55}$$

and  $\Omega_n$  is the volume of the unit *n*-sphere  $S^n$ . One may verify by using the expressions for the Ricci tensor  $\mathbb{R}^{MN}$  and Ricci scalar  $\mathbb{R}$  in terms of A and B [150] that all the field equations are reduced to a single equation (6.55).

Having established that the supergravity configuration preserves half the supersymmetries, we must also verify that the string configuration also preserve these supersymmetries. As discussed in [83], the criterion is that in addition to the existence of Killing spinors we must also have

$$(1 - \Gamma)\varepsilon = 0, \tag{6.56}$$

where the choice of sign is dictated by the choice of the sign in the Wess-Zumino term in  $S_2$ , and where

$$\Gamma \equiv \frac{1}{2!\sqrt{-\gamma}} \,\varepsilon^{ij} \partial_i X^M \partial_j X^N \Gamma_{MN}. \tag{6.57}$$

Since  $\Gamma^2 = 1$  and tr  $\Gamma = 0$ ,  $\frac{1}{2}(1 \pm \Gamma)$  act as projection operators. For our solution, we find that

$$\Gamma = \gamma_3 \otimes 1, \tag{6.58}$$

and hence (6.57) is satisfied. This explains, from a string point of view, why the solutions we have found preserve just half the supersymmetries. It originates from the fermionic kappa symmetry of the superstring action. The fermionic zero-modes on the worldvolume are just the Goldstone fermions associated with the broken supersymmetry. As shown in [97], the elementary string solution saturates a Bogolmol'nyi bound for the mass per unit length

$$\mathcal{M}_2 = \int d^8 y \ \theta_{00}, \tag{6.59}$$

where  $\theta_{MN}$  is the total energy-momentum pseudotensor of the combined gravity-matter system. One finds

$$\kappa \mathcal{M}_2 \ge \frac{1}{\sqrt{2}} |e_2| e^{\phi_0/2},$$
(6.60)

where  $e_2$  is the Noether "electric" charge whose conservation follows the equation of motion of the 2-form, namely

$$e_2 = \frac{1}{\sqrt{2\kappa}} \int_{S^7} e^{-\phi *} H, \tag{6.61}$$

where \* denotes the Hodge dual using the canonical metric and the integral is over an asymptotic seven-sphere surrounding the string. We find for our solution that

$$\mathcal{M}_2 = e^{\phi_0/2} T_2, \tag{6.62}$$

and

$$e_2 = \sqrt{2}\kappa \ T_2. \tag{6.63}$$

Hence the bound is saturated. This provides another way, in addition to unbroken supersymmetry, to understand the stability of the solution.

The elementary string discussed above is a solution of the coupled field-string system with action  $I_{10}(\text{string}) + S_2$ . As such it exhibits  $\delta$ -function singularities at y = 0. It is characterized by a non-vanishing Noether electric charge  $e_2$ . By contrast, we now wish to find a solitonic fivebrane, corresponding to a solution of the source free equations resulting from  $I_{10}(\text{string})$  alone and which will be characterized by a non-vanishing topological "magnetic" charge  $g_6$ .

To this end, we now make an ansatz invariant under  $P_6 \times SO(4)$ . Hence we write (6.22) and (6.23) as before where now  $\mu = 0, 1...5$  and m = 6, 7, 8, 9. The ansatz for the antisymmetric tensor, however, will now be made on the field strength rather than on the potential. From section (6.2) we recall that a non-vanishing electric charge corresponds to

$$\frac{1}{\sqrt{2\kappa}}e^{-\phi*}H = e_2\varepsilon_7/\Omega_7,\tag{6.64}$$

where  $\varepsilon_n$  is the volume form on  $S^n$ . Accordingly, to obtain a non-vanishing magnetic charge, we make the ansatz

$$\frac{1}{\sqrt{2\kappa}}H = g_6\varepsilon_3/\Omega_3. \tag{6.65}$$

Since this is an harmonic form, H can no longer be written globally as the curl of B, but it satisfies the Bianchi identity. It is now not difficult to show that all the field equations are satisfied. The solution is given by

$$e^{2\phi} = e^{2\phi_0} \left( 1 + \frac{k_6}{y^2} \right), \tag{6.66}$$

$$ds^{2} = e^{-(\phi - \phi_{0})/2} \eta_{\mu\nu} dx^{\mu} dx^{\nu} + e^{3(\phi - \phi_{0})/2} \delta_{mn} dy^{m} dy^{n}, \qquad (6.67)$$

$$H = 2k_6 e^{\phi_0/2} \varepsilon_3, \tag{6.68}$$

where  $\mu, \nu = 0, 1, ..., 5, m, n = 6, 7, 8, 9$  and where

$$k_6 = \frac{\kappa g_6}{\sqrt{2}\Omega_3} e^{-\phi_0/2}.$$
 (6.69)

It follows that the mass per unit 5-volume now saturates a bound involving the magnetic charge

$$\kappa \mathcal{M}_6 = \frac{1}{\sqrt{2}} \mid g_6 \mid e^{-\phi_0/2}.$$
(6.70)

Note that the  $\phi_0$  dependence is such that  $\mathcal{M}_6$  is large for small  $\mathcal{M}_2$  and vice-versa.

The electric charge of the elementary solution and the magnetic charge of the soliton solution obey a Dirac quantization rule [33, 40]

$$e_2 g_6 = 2\pi n, \qquad n = \text{integer}, \tag{6.71}$$

and hence

$$g_6 = 2\pi n / \sqrt{2}\kappa T_2. \tag{6.72}$$

# 6.3 The elementary fivebrane and solitonic string

In keeping with the viewpoint that the fivebrane may be regarded as fundamental in its own right, Duff and Lu [103] then constructed the *elementary fivebrane* solution by coupling the 7-form version of supergravity to a fivebrane  $\sigma$ -model source in analogy with the elementary string. This carries an electric charge  $e_6$ . Thus the elementary fivebrane, as pointed out by Callan, Harvey and Strominger [106, 108], could also be regarded as a soliton when viewed from the dual perspective, with  $g_6 = e_6$ . In other words, it provides a nonsingular solution of the source-free 3-form equations. By the same token, when viewed from the dual perspective, the elementary string provides a nonsingular solution of the source-free 7-form equations with  $g_2 = e_2$  [109].

	Elementary fivebrane	Solitonic fivebrane
metric	$e^{-\phi/6}g_{MN}(canon)$	$e^{\phi/2}g_{MN}(canon)$
action	$I_{10}(fivebrane) + S_6(fivebrane)$	$I_{10}(string) + S_{YM}(string)$
sources	$X_4 = e_6 \epsilon_4 \delta^4(y)$	$X_4 = g_6 T r F^2$
$T_{\mu\nu}$	$-T_6 e^{-\phi/2} g_{\mu\nu} \delta^4(y)$	$\tilde{T}_6 g_{\mu\nu} e^{-\phi/2} Tr F^2$
tension	$\sqrt{2}\kappa T_6 = e_6$	$\sqrt{2}\kappa\tilde{T}_6 = g_6$
charge	electric	magnetic
	$\sqrt{2}\kappa e_6 = \int\limits_{S^3} e^{\phi *} H_7$	$\sqrt{2}\kappa g_6 = \int\limits_{S^3} H_3$
mass	$\sqrt{2}\kappa\mathcal{M}_6 =  e_6 e^{-\phi_0/2}$	$\sqrt{2}\kappa\mathcal{M}_6= g_6 e^{-\phi_0/2}$

Table 6: Fivebranes: String/fivebrane duality implies  $e_6g_2 = 2\pi n = 2\kappa^2 T_6 \tilde{T}_2$  and identifying elementary and solitonic fivebranes yields  $e_6 = g_6$ .

	Elementary string	Solitonic string
metric	$e^{\phi/2}g_{MN}(canon)$	$e^{-\phi/6}g_{MN}(canon)$
action	$I_{10}(string) + S_2(string)$	$I_{10}(fivebrane) + S_{YM}(fivebrane)$
sources	$X_8 = e_2 \epsilon_8 \delta^8(y)$	$X_8 = g_2 T r F^4$
$T_{\mu\nu}$	$-T_2 e^{\phi/2} g_{\mu\nu} \delta^8(y)$	$ ilde{T}_2 g_{\mu u} e^{\phi/2} Tr F^4$
tension	$\sqrt{2}\kappa T_2 = e_2$	$\sqrt{2\kappa}\tilde{T}_2 = g_2$
charge	electric	magnetic
	$\sqrt{2}\kappa e_2 = \int\limits_{S^7} e^{-\phi *} H_3$	$\sqrt{2}\kappa g_2 = \int\limits_{S^7} H_7$
mass	$\sqrt{2}\kappa\mathcal{M}_2 =  e_2 e^{\phi_0/2}$	$\sqrt{2}\kappa\mathcal{M}_2 =  g_2 e^{\phi_0/2}$

Table 7: Strings: String/fivebrane duality implies  $e_2g_6 = 2\pi n = 2\kappa^2 T_2 \tilde{T}_6$  and identifying elementary and solitonic strings yields  $e_2 = g_2$ .

## 6.4 Honey, I shrunk the instanton

Now we incorporate the Yang-Mills fields. In the case of Strominger's solitonic fivebrane

$$X_4 = \frac{1}{30\pi T_2} TrF \wedge F \tag{6.73}$$

String/fivebrane duality then suggested that by coupling the 7-form version of supergravity to super Yang-Mills (without a  $\sigma$ -model source), one ought to find a nonsingular heterotic string soliton carrying a topological magnetic charge  $g_2$ . This was indeed the case [109], but scaling arguments required an unconventional Yang-Mills Lagrangian, quartic in the field strengths with

$$X_8 = \frac{1}{3(2\pi)^3 T_6} \left( TrF \wedge F \wedge F \wedge F - \frac{1}{7200} TrF \wedge F \wedge TrF \wedge F \right)$$
(6.74)

The fivebrane equations admit a solution where  $F_{mn}$  is a self-dual SO(4) instanton [4] in the 4 directions orthogonal to the brane. Similarly, the string equations admit a solution where  $F_{mn}$  is an SO(8) instanton [78] in the 8 directions orthogonal to the string.

Consistency demands that the sources, denoted generically by J, must be such that when we shrink the size of the instanton

$$\lim \rho \to 0 \quad J(quadratic \, Yang - Mills) = J(fivebrane \, sigma \, model) \tag{6.75}$$

$$\lim \rho \to 0 \quad J(quartic Yang - Mills) = J(string sigma model) \tag{6.76}$$

This is indeed the case. For example the corresponding dilaton solutions (with  $\phi_0 = 0$ ) are

$$e^{-2\phi} = 1 + \frac{k_6(y^2 + 2\rho^2)}{(y^2 + \rho^2)^2} \to 1 + \frac{k_6}{y^2}$$
 (6.77)

for the 5-brane and

$$e^{-2\phi} = 1 + k_2 \frac{(y^6 + 6y^4\rho^2 + 15y^2\rho^4 + 20\rho^6)}{(y^2 + \rho^2)^6} \to 1 + \frac{k_2}{y^6}$$
(6.78)

for the string.

#### 6.5 Message to the no-braners

Let me reiterate. This discovery of solitons means that 5-branes are there whether you like them or not. If you buy strings, and of course you are free not to, you have to buy 5-branes in the same package.

# 6.6 N=4 gauge theory on the D3-brane

We have recently constructed a self-dual Type IIB super 3-brane which represents a new point (d = 4, D = 10) on the brane-scan. Earlier no-go theorems [58] are circumvented because there are spin 1 fields on the worldvolume. In fact, the gauge-fixed theory on the worldvolume is described by a (d = 4, N = 4) Maxwell multiplet.

### 6.7 Subsequent developments

• Branes and M-theory

The realization that the equations of string theory admit branes as soliton solutions opened a new window on non-perturbative string theory and paved the way for M-theory.

• Fivebranes

The 5-branes discussed in this section now feature prominently in M-theory, for example heterotic/heterotic duality [185]. They became known as Neveu-Schwarz 5-branes.

• Dual frame metrics

The string and fivebrane  $\sigma$ -model metrics in D = 10 are special cases of (d-1)brane metrics in D dimensions and those of their  $\tilde{d} = D - 2 - d$  duals.

$$g_{MN}(d) = e^{a(d)\phi/d}g_{MN}(canon)$$
  

$$g_{MN}(\tilde{d}) = e^{a(d)\phi/\tilde{d}}g_{MN}(canon)$$
(6.79)

where  $a(\tilde{d}) = -a(d)$  and

$$a^2(d) = 4 - \frac{2dd}{d+\tilde{d}} \tag{6.80}$$

These dual frame metrics, for which by definition the Nambu-Goto part of the (d-1)-brane sigma model is independent of the dilaton, have found numerous applications, especially in the context of holography [145, 239, 245, 306, 360].

• Type II branes

According to the classification of [58] described in Section 3, no Type II *p*-branes with p > 1 could exist. Moreover, the only brane allowed in D = 11 was p = 2. These conclusions were based on the assumption that the only fields propagating on the worldvolume were scalars and spinors, so that, after gauge fixing, they fall only into *scalar* supermultiplets, denoted by s on the brane-scan of Table 1. Indeed, these were the only kappa symmetric actions known at the time. However, as we saw already in Section 4.1, there was evidence for a Type IIB self-dual 3-brane and an M5-brane [68]. Moreover, using soliton arguments, it was pointed out in [106, 108] that both Type IIA and Type IIB superfivebranes exist after all. Moreover, the Type IIB theory also admits the self-dual superthreebrane as a soliton [112]. The no-go theorem is circumvented because in addition to the superspace coordinates  $X^M$  and  $\theta^{\alpha}$  there are also higher spin fields on the worldvolume: vectors or antisymmetric tensors. This raised the question: are there other super p-branes and if so, for what p and D? In [124] an attempt was made to answer this question by asking what new points on the brane-scan are permitted by bose-fermi matching alone. Given that the gauge-fixed theories display worldvolume supersymmetry, and given that we now wish to include the possibility of vector and antisymmetric tensor fields, it is a relatively straightforward exercise to repeat the bose-fermi matching conditions of the Section 3 for vector and antisymmetric tensor supermultiplets. Let us begin with vector supermultiplets. Once again, we may proceed in one of two ways. First, given that a worldvolume vector has (d-2) degrees of freedom, the scalar multiplet condition (3.25) gets replaced by

$$D - 2 = \frac{1}{2} mn = \frac{1}{4} MN.$$
 (6.81)

Alternatively, we may simply list all the vector supermultiplets in the classification of [48] and once again interpret D via (3.27). The results [124, 150] are shown by the points labelled v in Table 1. Next we turn to antisymmetric tensor multiplets. In d = 6 there is a supermultiplet with a second rank tensor whose field strength is self-dual:  $(B_{\mu\nu}^-, \lambda^I, \phi^{[IJ]}), I = 1, \ldots, 4$ . This is has chiral d = 6 supersymmetry. Since there are five scalars, we have D = 6+5 = 11. There is thus a new point on the scan corresponding to the D = 11 superfivebrane. One may decompose this  $(n_+, n_-) = (2, 0)$  supermultiplet under  $(n_+, n_-) = (1, 0)$  into a tensor multiplet with one scalar and a hypermultiplet with four scalars. Truncating to just the tensor multiplet gives the zero modes of a fivebrane in D = 6 + 1 = 7. These two tensor multiplets are shown by the points labelled t in Table 1.

• D-branes

Subsequently, all the v-branes were given a new interpretation as Dirichlet p-

branes, called D-branes, surfaces of dimension p on which open strings can end and which carry R-R (Ramond-Ramond) charge [168]. The IIA theory has Dbranes with p = 0, 2, 4, 6, 8 and the IIB theory has D-branes with p = 1, 3, 5, 7, 9. They are related to one another by T-duality. In terms of how their tensions depend on the string coupling  $g_s$ , the D-branes are mid-way between the fundamental (F) strings and the solitonic (S) fivebranes:

$$T_{F1} \sim m_s^2$$
,  $T_{Dp} \sim \frac{m_s^{p+1}}{g_s}$ ,  $T_{S5} \sim \frac{m_s^6}{g_s^2}$  (6.82)

• M2-branes, M5-branes and the quantization of 4-form flux

That 4-form flux of M-theory is quantized was implicit in the multimembrane solution of D = 11 supergravity since the tension of a stack of N 2-branes is just N times that of a single brane  $T_3$ . It was spelled out explicitly in [160] where we begin with the bosonic sector of the d = 3 worldvolume of the D = 11supermembrane (3.10) and the bosonic D = 11 supergravity action (3.11) While there are two dimensionful parameters, the membrane tension  $T_3$  and the elevendimensional gravitational constant  $\kappa_{11}$ , they are in fact not independent. To see this, we note from (3.10) that  $A_3$  has period  $2\pi/T_3$  so that  $F_4$  is quantized according to

$$\int F_4 = \frac{2\pi n}{T_3} \qquad n = \text{integer} .$$
(6.83)

Consistency of such  $A_3$  periods with the spacetime action, (3.11), gives the relation<sup>12</sup>

$$\frac{(2\pi)^2}{2\kappa_{11}^2 T_3^3} \in Z \ . \tag{6.84}$$

The D = 11 classical field equations admit as a soliton a dual superfivebrane [118, 134] which couples to the dual field strength  $\tilde{F}_7 = *F_4$ . The fivebrane tension  $\tilde{T}_6$  is given by the Dirac quantization rule [134]

$$2\kappa_{11}^2 T_3 \tilde{T}_6 = 2\pi n \qquad n = \text{integer} .$$
 (6.85)

Using (6.84), this may also be written as

$$2\pi \tilde{T}_6 = T_3^{\ 2} \tag{6.86}$$

Although Dirac quantization rules of the type (6.85) appear for other *p*-branes and their duals in lower dimensions [134], it is the absence of a dilaton in the

 $<sup>^{12}\</sup>mathrm{A}$  factor of 2 error in [160] was corrected in [169, 204]

D = 11 theory that allows us to fix both the gravitational constant and the dual tension in terms of the fundamental tension.

From (3.12), the fivebrane Bianchi identity reads

$$d\tilde{F}_7 = -\frac{1}{2}F_4^2 \ . \tag{6.87}$$

However, such a Bianchi identity will in general require gravitational Chern-Simons corrections arising from a sigma-model anomaly on the fivebrane worldvolume [125, 121, 119, 142, 49, 213, 146]:

$$d\tilde{F}_7 = -\frac{1}{2}F_4{}^2 + (2\pi)^4 \tilde{\beta}' \tilde{X}_8 , \qquad (6.88)$$

where  $\tilde{\beta}'$  is related to the fivebrane tension by  $T_6 = 1/(2\pi)^3 \tilde{\beta}'$  and where the 8-form polynomial  $\tilde{X}_8$ , quartic in the gravitational curvature R, describes the  $d = 6 \sigma$ -model Lorentz anomaly of the D = 11 fivebrane.

$$\tilde{X}_8 = \frac{1}{(2\pi)^4} \left[ -\frac{1}{768} (\mathrm{tr}R^2)^2 + \frac{1}{192} \mathrm{tr}R^4 \right] \,. \tag{6.89}$$

Thus membrane/fivebrane duality predicts a spacetime correction to the D = 11 supermembrane action

$$I_{11}(\text{Lorentz}) = T_3 \int A_3 \wedge \frac{1}{(2\pi)^4} \Big[ -\frac{1}{768} (\text{tr}R^2)^2 + \frac{1}{192} \text{tr}R^4 \Big] .$$
 (6.90)

By simultaneous dimensional reduction [50] of (d = 3, D = 11) to (d = 2, D = 10)on  $S^1$ , this prediction translates into a corresponding prediction for the Type IIA string:

$$I_{10}(\text{Lorentz}) = T_2 \int B_2 \wedge \frac{1}{(2\pi)^4} \left[ -\frac{1}{768} (\text{tr}R^2)^2 + \frac{1}{192} \text{tr}R^4 \right], \qquad (6.91)$$

where  $B_2$  is the string 2-form,  $T_2$  is the string tension,  $T_2 = 1/2\pi\alpha'$ , related to the membrane tension by

$$T_2 = 2\pi R T_3 , \qquad (6.92)$$

where R is the  $S^1$  radius.

Further elaboration of four-form flux quantization may be found in [208, 201]

• D-branes from M-branes

In addition to M2 and M5 there are two other objects in D = 11, the plane wave [28] and the Kaluza-Klein monopole [34], which though not branes are still 1/2 BPS. When spacetime is compactified a *p*-brane may remain a *p*-brane or else become a (p-k)-brane if it wraps around k of the compactified directions. For example, the Type IIA fundamental string emerges by wrapping the M2-brane around  $S^1$  and shrinking its radius to zero, and the Type IIA 4-brane emerges in a similar way from the M5-brane. Several comments are now in order: (1) The number of scalars in a vector supermultiplet is such that, from (3.27), D = 3, 4, 6or 10 only, in accordance with [48]. (2) Vector supermultiplets exist for all  $d \leq 10$ [48], as may be seen by dimensionally reducing the (n = 1, d = 10) Maxwell supermultiplet. However, in d = 2 vectors have no degrees of freedom and in d = 3vectors have only one degree of freedom and are dual to scalars. In this sense, therefore, these multiplets will already have been included as scalar multiplets in Section 3. There is consequently some arbitrariness in whether we count these as new points on the scan. For example, it was recognized [124] that by dualizing a vector into a scalar on the gauge-fixed d = 3 worldvolume of the Type IIA supermembrane, one increases the number of worldvolume scalars, *i.e.* transverse dimensions, from 7 to 8 and hence obtains the corresponding worldvolume action of the D = 11 supermembrane. Thus the D = 10 Type IIA theory contains a hidden D = 11 Lorentz invariance [124, 178]! More on D-branes from M-branes may be found in the paper by Townsend [178]. (3) In listing vector multiplets, we have focussed only on the abelian theories obtained by dimensionally reducing the Maxwell multiplet. One might ask what role, if any, is played by non-abelian Yang-Mills multiplets.

• Non-abelian gauge groups from stacked branes

Since they are BPS, there is a no-force condition between the branes that allows us to have many branes of the same charge parallel to one another. The gauge group on a single D-brane is an abelian U(1). If we stack N such branes on top of one another, the gauge group is the non-abelian U(N). As we separate them this decomposes into its subgroups, so in fact there is a Higgs mechanism at work whereby the vacuum expectation values of the Higgs fields are related to the separation of the branes. For example the theory that lives on a stack of N Type IIB D3 branes is a four-dimensional U(N) n = 4 super Yang-Mills theory. In the limit of large N the geometry of this configuration tends to the product of five dimensional anti-de Sitter space and a five dimensional sphere,  $AdS_5 \times S^5$ . This provides the AdS/CFT correspondence.

• The brane-world

The 3-brane soliton of Type IIB supergravity was an early candidate for a 'braneworld', firstly because of its dimensionality [107, 112] and secondly because gauge fields propagate on its worldvolume [112]. The idea that our universe is a brane floating in a higher dimensional bulk is not new. See, for example [23] and [61]. But the way in which the gauge fields are confined to the brane in the D-brane picture and in the Horava-Witten [172, 193] heterotic M-theory picture provided the impetus for a revival of the *braneworld* and large extra dimensions [233, 234, 247, 248, 262].

# 7 1992 Four-dimensional string/string duality

26th Workshop of the Eloisatron Project, Erice, Italy, December 5-12, 1992 [133]

In this lecture we presented supersymmetric soliton solutions of the four-dimensional heterotic string corresponding to monopoles, strings and domain walls. These solutions admit the D = 10 interpretation of a fivebrane wrapped around 5, 4 or 3 of the 6 toroidally compactified dimensions and are arguably exact to all orders in  $\alpha'$ . The solitonic string solution exhibits an SL(2, Z) strong/weak coupling duality which however corresponds to an SL(2, Z) target space duality of the fundamental string.

## 7.1 Fivebranes in D=10

We first summarize the 't Hooft ansatz for the Yang-Mills instanton. Consider the four-dimensional Euclidean action

$$S = -\frac{1}{2g^2} \int d^4 x \operatorname{Tr} F_{\mu\nu} F^{\mu\nu}, \qquad \mu, \nu = 1, 2, 3, 4.$$
 (7.1)

For gauge group SU(2), the fields may be written as  $A_{\mu} = (g/2i)\sigma^a A^a_{\mu}$  and  $F_{\mu\nu} = (g/2i)\sigma^a F^a_{\mu\nu}$  (where  $\sigma^a$ , a = 1, 2, 3 are the 2 × 2 Pauli matrices). A self-dual solution (but not the most general one) to the equation of motion of this action is given by the 't Hooft ansatz

$$A_{\mu} = i\overline{\Sigma}_{\mu\nu}\partial_{\nu}\ln f \tag{7.2}$$

where  $\overline{\Sigma}_{\mu\nu} = \overline{\eta}^{i\mu\nu}(\sigma^i/2)$  for i = 1, 2, 3, where  $\overline{\eta}^{i\mu\nu} = -\overline{\eta}^{i\nu\mu} = \epsilon^{i\mu\nu}$  for  $\mu, \nu = 1, 2, 3$ , and  $\overline{\eta}^{i\mu\nu} = -\overline{\eta}^{i\nu\mu} = -\delta^{i\mu}$  for  $\nu = 4$  and where  $f^{-1} \Box f = 0$ . The ansatz for the anti-selfdual solution is similar, with the  $\delta$ -term changing sign. From this ansatz, depending on how many of the four coordinates f is allowed to depend and depending on whether we compactify, we shall obtain D = 10 multi-fivebrane and D = 4 multi-monopole, multi-string and multi-domain wall solutions. We will discuss these four cases in the next section. In this section, we do not specify the precise form of f or the dilaton function, but show that the derivation of the solution and most of the arguments used to demonstrate the exactness of the heterotic solution are equally valid for any fsatisfying  $f^{-1} \Box f = 0$ .

It turns out that there is an analog to the 't Hooft ansatz for the Yang-Mills instanton in the gravitational sector of the string, namely the axionic instanton [114]. In its simplest form, this instanton appears as a solution for the massless fields of the bosonic string. The identical instanton structure arises in all supersymmetric multifivebrane solutions, in particular in the tree-level neutral solution

$$g_{\mu\nu} = e^{2\phi} \delta_{\mu\nu} \qquad \mu, \nu = 1, 2, 3, 4,$$
  

$$g_{ab} = \eta_{ab} \qquad a, b = 0, 5, ..., 9,$$
  

$$H_{\mu\nu\lambda} = \pm 2\epsilon_{\mu\nu\lambda\sigma} \partial^{\sigma} \phi \qquad \mu, \nu, \lambda, \sigma = 1, 2, 3, 4,$$
(7.3)

with  $e^{-2\phi} \square e^{2\phi} = 0$ . The D'Alembertian refers to the four-dimensional subspace  $\mu, \nu, \lambda, \sigma = 1, 2, 3, 4$  and  $\phi$  is taken to be independent of  $(x^0, x^5, x^6, x^7, x^8, x^9)$ . For zero background fermionic fields the above solution breaks half the spacetime supersymmetries.

The generalized curvature of this solution was shown [120, 137] to possess (anti) self-dual structure similar to that of the 't Hooft ansatz. To see this we define a generalized curvature  $\hat{R}^{\mu}{}_{\nu\rho\sigma}$  in terms of the standard curvature  $R^{\mu}{}_{\nu\rho\sigma}$  and  $H_{\mu\alpha\beta}$ :

$$\hat{R}^{\mu}{}_{\nu\rho\sigma} = R^{\mu}{}_{\nu\rho\sigma} + \frac{1}{2} \left( \nabla_{\sigma} H^{\mu}{}_{\nu\rho} - \nabla_{\rho} H^{\mu}{}_{\nu\sigma} \right) + \frac{1}{4} \left( H^{\lambda}{}_{\nu\rho} H^{\mu}{}_{\sigma\lambda} - H^{\lambda}{}_{\nu\sigma} H^{\mu}{}_{\rho\lambda} \right).$$
(7.4)

One can also define  $\hat{R}^{\mu}{}_{\nu\rho\sigma}$  as the Riemann tensor generated by the generalized Christoffel symbols  $\hat{\Gamma}^{\mu}{}_{\alpha\beta}$ , where  $\hat{\Gamma}^{\mu}{}_{\alpha\beta} = \Gamma^{\mu}{}_{\alpha\beta} - (1/2)H^{\mu}{}_{\alpha\beta}$ . The crucial observation for obtaining higher-loop and even exact solutions is the following. For any solution given by (7.3), we can express the generalized curvature in terms of the dilaton field as

$$\hat{R}^{\mu}{}_{\nu\rho\sigma} = \delta_{\mu\sigma} \nabla_{\rho} \nabla_{\nu} \phi - \delta_{\mu\rho} \nabla_{\sigma} \nabla_{\nu} \phi + \delta_{\nu\rho} \nabla_{\sigma} \nabla_{\mu} \phi - \delta_{\nu\sigma} \nabla_{\rho} \nabla_{\mu} \phi \pm \epsilon_{\mu\nu\rho\lambda} \nabla_{\sigma} \nabla_{\lambda} \phi \mp \epsilon_{\mu\nu\sigma\lambda} \nabla_{\rho} \nabla_{\lambda} \phi.$$

$$(7.5)$$

It easily follows that

$$\hat{R}^{\mu}{}_{\nu\rho\sigma} = \mp \frac{1}{2} \epsilon_{\rho\sigma}{}^{\lambda\gamma} \hat{R}^{\mu}{}_{\nu\lambda\gamma}.$$
(7.6)

So the (anti) self-duality appears in the gravitational sector of the string in terms of its generalized curvature.

We now turn to the exact heterotic solution. The tree-level supersymmetric vacuum equations for the heterotic string are given by

$$\delta\psi_M = \left(\nabla_M - \frac{1}{4}H_{MAB}\Gamma^{AB}\right)\epsilon = 0 \tag{7.7}$$

$$\delta\lambda = \left(\Gamma^A \partial_A \phi - \frac{1}{6} H_{ABC} \Gamma^{ABC}\right) \epsilon = 0 \tag{7.8}$$

$$\delta\chi = F_{AB}\Gamma^{AB}\epsilon = 0 \tag{7.9}$$

where A, B, C, M = 0, 1, 2, ..., 9 and where  $\psi_M$ ,  $\lambda$  and  $\chi$  are the gravitino, dilatino and gaugino fields. The Bianchi identity is given by

$$dH = \frac{\alpha'}{4} \left( \text{tr}R \wedge R - \frac{1}{30} \text{Tr}F \wedge F \right).$$
(7.10)

The (9 + 1)-dimensional Majorana-Weyl fermions decompose into chiral spinors according to  $SO(9,1) \supset SO(5,1) \otimes SO(4)$  for the  $M^{9,1} \to M^{5,1} \times M^4$  decomposition. Then (7.3) with arbitrary dilaton and with constant chiral spinors  $\epsilon_{\pm}$  solves the supersymmetry equations with zero background fermi fields provided the YM gauge field satisfies the instanton (anti) self-duality condition [98]

$$F_{\mu\nu} = \pm \frac{1}{2} \epsilon_{\mu\nu}{}^{\lambda\sigma} F_{\lambda\sigma}.$$
(7.11)

In the absence of a gauge sector, the multi-fivebrane solution is identical to the "neutral" tree-level solution shown in (7.3). A perturbative "gauge" fivebrane solution was found in [98] An exact solution is obtained as follows. Define a generalized connection by

$$\Omega_{\pm M}^{AB} = \omega_M^{AB} \pm H_M^{AB} \tag{7.12}$$

in an SU(2) subgroup of the gauge group, and equate it to the gauge connection  $A_{\mu}$ [5, 6] so that the corresponding curvature  $R(\Omega_{\pm})$  cancels against the Yang-Mills field strength F and dH = 0. For  $e^{-2\phi} \square e^{2\phi} = 0$  (or  $e^{2\phi} = e^{2\phi_0} f$ ) the curvature of the generalized connection can be written in terms of the dilaton as in from which it follows that both F and R are (anti) self-dual. This solution becomes exact since  $A_{\mu} = \Omega_{\pm\mu}$  implies that all the higher order corrections vanish [106, 108]. The self-dual solution for the gauge connection is then given by the 't Hooft ansatz. So the heterotic solution combines a YM instanton in the gauge sector with an axionic instanton in the gravity sector. In addition, the heterotic solution has finite action. Further arguments supporting the exactness of this solution based on (4, 4) worldsheet supersymmetry are shown in [106]. Note that at no point in this discussion do we refer to the specific form of f, so that all of the above arguments apply for an arbitrary solution of  $f^{-1}\Box f = 0$ .

We now go back to the 't Hooft ansatz and solve the equation  $f^{-1}\square f = 0$ . If we take f to depend on all four coordinates we obtain a multi-instanton solution

$$f_I = 1 + \sum_{i=1}^{N} \frac{\rho_i^2}{|\vec{x} - \vec{a}_i|^2}$$
(7.13)

where  $\rho_i^2$  is the instanton scale size and  $\vec{a}_i$  the location in four-space of the *i*th instanton. For  $e^{2\phi} = e^{2\phi_0} f_I$ , and assuming no dimensions are compactified, we obtain from (7.3) the neutral fivebrane of [104] and the exact heterotic fivebrane of [106, 108] in D = 10. The solitonic fivebrane tension  $\widetilde{T}_6$  is related to the fundamental string tension  $T_2$  (=  $1/2\pi\alpha'$ ) by the Dirac quantization condition

$$\kappa_{10}^2 \widetilde{T}_6 T_2 = n\pi \tag{7.14}$$

where n is an integer and where  $\kappa_{10}^2$  is the D = 10 gravitational constant. This implies  $\rho_i^2 = e^{-2\phi_0} n_i \alpha'$ , where  $n_i$  are integers. Near each source the solution is described by an exact conformal field theory [106, 108, 114, 120].

# 7.2 Monopoles, strings and domain walls in D=4

Instead, let us single out a direction in the transverse four-space (say  $x^4$ ) and assume all fields are independent of this coordinate. Since all fields are already independent of  $x^5, x^6, x^7, x^8, x^9$ , we may consistently assume the  $x^4, x^5, x^6, x^7, x^8, x^9$  are compactified on a six-dimensional torus, where we shall take the  $x^4$  circle to have circumference  $Le^{-\phi_0}$  and the rest to have circumference L, so that  $\kappa_4^2 = \kappa_{10}^2 e^{\phi_0}/L^6$ . Then the solution for f satisfying  $f^{-1}\square f = 0$  has multi-monopole structure.

We may then modify the solution of the 't Hooft ansatz even further and choose two directions in the four-space (1234) (say  $x^3$  and  $x^4$ ) and assume all fields are independent of both of these coordinates. We may now consistently assume that  $x^3, x^4, x^6, x^7, x^8, x^9$ are compactified on a six-dimensional torus, where we shall take the  $x^3$  and  $x^4$  circles to have circumference  $Le^{-\phi_0}$  and the remainder to have circumference L, so that  $\kappa_4^2 = \kappa_{10}^2 e^{2\phi_0}/L^6$ . Then the solution for f satisfying  $f^{-1} \Box f = 0$  has multi-string structure

We complete the family of solitons that can be obtained from the solutions of the 't Hooft ansatz by demanding that f depend on only one coordinate, say  $x^1$ . We may now consistently assume that  $x^2, x^3, x^4, x^7, x^8, x^9$  are compactified on a six-dimensional torus, where we shall take the  $x^2, x^3$  and  $x^4$  circles to have circumference  $Le^{-\phi_0}$  and the rest to have circumference  $\kappa_4^2 = \kappa_{10}^2 e^{3\phi_0}/L^6$ . Then the solution of  $f^{-1} \Box f = 0$  has domain wall structure.

As for the fivebrane in D = 10, the mass of the monopole, the mass per unit length of the string and the mass per unit area of the domain wall saturate a Bogomol'nyi bound with the topological charge. (In the case of the string and domain, wall, however, we must extrapolate the meaning of the ADM mass to non-asymptotically flat spacetimes.)

# 7.3 String/string duality

Let us focus on the solitonic string configuration in the case of a single source. In terms of the complex field

$$T = T_1 + iT_2 = B_{34} + ie^{-2\sigma} = B_{34} + i\sqrt{\det g_{mn}^S} \qquad m, n = 3, 4, 6, 7, 8, 9,$$
(7.15)

where  $g_{MN}^S$  is the string  $\sigma$ -model metric, the solution takes the form (with  $z = x_1 + x_2$ )

$$T = \frac{1}{2\pi i} \ln \frac{z}{r_0}$$
(7.16)

$$ds^{2} = -dt^{2} + dx_{5}^{2} - \frac{1}{2\pi} \ln \frac{r}{r_{0}} dz d\overline{z}$$
(7.17)

whereas both the four-dimensional (shifted) dilaton  $\eta = \phi + \sigma$  and the four-dimensional two-form  $B_{\mu\nu}$  are zero. In terms of the canonical metric  $g_{\mu\nu}$ ,  $T_1$  and  $T_2$ , the relevant part of the action takes the form

$$S_4 = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} \left( R - \frac{1}{2T_2^2} g^{\mu\nu} \partial_\mu T \partial_\nu \overline{T} \right)$$
(7.18)

and is invariant under the SL(2, R) transformation

$$T \to \frac{aT+b}{cT+d}, \qquad ad-bc=1.$$
 (7.19)

The discrete subgroup SL(2, Z), for which a, b, c and d are integers, is just a subgroup of the O(6, 22; Z) target space duality, which can be shown to be an exact symmetry of the compactified string theory at each order of the string loop perturbation expansion.

This SL(2, Z) is to be contrasted with the SL(2, Z) symmetry of the elementary four-dimensional solution of [97]. In their solution  $T_1$  and  $T_2$  are zero, but  $\eta$  and  $B_{\mu\nu}$ are non-zero. The relevant part of the action is

$$S_4 = \frac{1}{2\kappa_4^2} \int d^4x \sqrt{-g} \left( R - 2g^{\mu\nu}\partial_\mu\eta\partial_\nu\eta - \frac{1}{12}e^{-4\eta}H_{\mu\nu\rho}H^{\mu\nu\rho} \right).$$
(7.20)

The equations of motion of this theory also display an SL(2, R) symmetry, but this becomes manifest only after dualizing and introducing the axion field a via

$$\sqrt{-g}g^{\mu\nu}\partial_{\nu}a = \frac{1}{3!}\epsilon^{\mu\nu\rho\sigma}H_{\nu\rho\sigma}e^{-4\eta}.$$
(7.21)

Then in terms of the complex field

$$S_1 + iS_2 = a + ie^{-2\eta} \tag{7.22}$$

the Dabholkar et al. fundamental string solution may be written

$$S = \frac{1}{2\pi i} \ln \frac{z}{r_0}$$
(7.23)

$$ds^{2} = -dt^{2} + dx_{5}^{2} - \frac{1}{2\pi} \ln \frac{r}{r_{0}} dz d\overline{z}.$$
(7.24)

Thus the two solutions are the same with the replacement  $T \leftrightarrow S$ . It has been conjectured that this second SL(2, Z) symmetry may also be a symmetry of string theory [96, 122, 140], but this is far from obvious order by order in the string loop expansion since it involves a strong/weak coupling duality  $\eta \rightarrow -\eta$ . What interpretation are we to give to these two SL(2, Z) symmetries: one an obvious symmetry of the fundamental string and the other an obscure symmetry of the fundamental string?

# 7.4 Subsequent developments

• String-string duality in D = 6 implies S-duality in D = 4

S-duality in D = 4 gauge theories refers to the conjectured SL(2, Z) symmetry that acts on the gauge coupling constant e and theta angle  $\theta$ :

$$S \to \frac{aS+b}{cS+d} \tag{7.25}$$

where a, b, c, d are integers satisfying ad - bc = 1 and where

$$S = S_1 + iS_2 = \frac{\theta}{2\pi} + i\frac{4\pi}{e^2}$$
(7.26)

This is also called electric/magnetic duality because the integers m and n which characterize the magnetic charges  $Q_m = n/e$  and electric charges  $Q_e = e(m + n\theta/2\pi)$  of the particle spectrum transform as

$$\left(\begin{array}{c}m\\n\end{array}\right) \rightarrow \left(\begin{array}{c}a&b\\c&d\end{array}\right) \left(\begin{array}{c}m\\n\end{array}\right)$$
(7.27)

Such a symmetry would be inherently non-perturbative since, for  $\theta = 0$  and with a = d = 0 and b = -c = -1, it reduces to the strong/weak coupling duality

$$e^2/4\pi \to 4\pi/e^2$$
  
 $n \to m, m \to -n$  (7.28)

This in turn means that the coupling constant cannot get renormalized in perturbation theory and hence that the renormalization group  $\beta$ -function vanishes

$$\beta(e) = 0 \tag{7.29}$$

This is guaranteed in N = 4 supersymmetric Yang-Mills and also happens in certain N = 2 theories. Moreover electric-magnetic duality follows by embedding these Yang-Mills theories in a superstring theory. In string theory the roles of the theta angle  $\theta$  and coupling constant e are played by the VEVs of the the four-dimensional axion field a and dilaton field  $\eta$ :

$$\langle a \rangle = \frac{\theta}{2\pi}$$
 (7.30)

$$e^2/4\pi = \langle e^\eta \rangle = 8G/\alpha'$$
 (7.31)

Here G is Newton's constant and  $2\pi\alpha'$  is the inverse string tension. Hence Sduality (7.25) now becomes a transformation law for the axion/dilaton field S:

$$S = S_1 + iS_2 = \mathbf{a} + ie^{-\eta} \tag{7.32}$$

The S-duality conjecture in string theory has its origins in supergravity. In the late 70s and early 80s, it was realized that compactified supergravity theories exhibit non-compact global symmetries [8, 11, 31] e.g SL(2, R), O(22, 6), O(24, 8),  $E_7$ ,  $E_8$ ,  $E_9$ ,  $E_{10}$ . In 1990 it was conjectured [99, 100] that discrete subgroups of all these symmetries should be promoted to duality symmetries of either heterotic or Type II superstrings. The case for O(22, 6; Z) had already been made. This is the well-established target space duality, sometimes called T-duality [139]. Stronger evidence for a strong/weak coupling SL(2, Z) duality in string theory was subsequently provided in [96, 114, 116, 122, 126, 137, 138, 140, 150], stronger evidence for the combination of S and T-duality into an O(24, 8; Z) in heterotic strings was provided in [99, 144] and stronger evidence for their combination into a discrete  $E_7$  in Type II strings was provided in [148], where it was dubbed U-duality.

Let us first consider *T*-duality and focus just on the moduli fields that arise in compactification on a 2-torus of a D = 6 string with dilaton  $\Phi$ , metric  $G_{MN}$  and 2-form potential  $B_{MN}$  with 3-form field strength  $H_{MNP}$ . Here the *T*-duality is just O(2,2;Z). Let us parametrize the compactified (m, n = 4, 5) components of string metric and 2-form as

$$G_{mn} = e^{\rho - \sigma} \begin{pmatrix} e^{-2\rho} + c^2 & -c \\ -c & 1 \end{pmatrix}$$

$$(7.33)$$

and

$$B_{mn} = b\epsilon_{mn} \tag{7.34}$$

The four-dimensional shifted dilaton  $\eta$  is given by

$$e^{-\eta} = e^{-\Phi} \sqrt{det G_{mn}} = e^{-\Phi-\sigma} \tag{7.35}$$

and the axion field a is defined by

$$\epsilon^{\mu\nu\rho\sigma}\partial_{\sigma}a = \sqrt{-g}e^{-\eta}g^{\mu\sigma}g^{\nu\lambda}g^{\rho\tau}H_{\sigma\lambda\tau}$$
(7.36)

where  $g_{\mu\nu} = G_{\mu\nu}$  and  $\mu, \nu = 0, 1, 2, 3$ . We further define the complex Kahler form field T and the complex structure field U by

$$T = T_1 + iT_2 = b + ie^{-\sigma}$$
  
$$U = U_1 + iU_2 = c + ie^{-\rho}$$
(7.37)

Thus this T-duality may be written as

$$O(2,2;Z)_{TU} \sim SL(2,Z)_T \times SL(2,Z)_U$$
 (7.38)

where  $SL(2,Z)_T$  acts on the *T*-field and  $SL(2,Z)_U$  acts on the *U*-field in the same way that  $SL(2,Z)_S$  acts on the *S*-field in (7.25). In contrast to  $SL(2,Z)_S$ ,  $SL(2,Z)_T \times SL(2,Z)_U$  is known to be not merely a symmetry of the supergravity theory but an exact string symmetry order by order in string perturbation theory.  $SL(2,Z)_T$  does, however, contain a minimum/maximum length duality mathematically similar to (7.28)

$$R \to \alpha'/R$$
 (7.39)

where R is the compactification scale given by

$$\alpha'/R^2 = \langle e^{\sigma} \rangle . \tag{7.40}$$

Even before compactification, the Type IIB supergravity exhibits an SL(2, R)whose discrete subgroup has been conjectured to be a non-perturbative symmetry of the Type IIB string [108, 148]. We shall refer to this duality as  $SL(2, Z)_X$  to distinguish it from the others. Combining this with the known *T*-duality of the four dimensional theory obtained by compactification on  $T^6$  leads to the  $E_7$ . So the explanation for *U*-duality devolves upon the explanation for this  $SL(2, Z)_X$ . Let us now investigate how both N = 4 and N = 2 exact electric/magnetic duality follows from string theory. As discussed above, there is a formal similarity between this symmetry and that of *T*-duality. It was argued in [152] that these mathematical similarities between  $SL(2, Z)_S$  and  $SL(2, Z)_T$  are not coincidental. Evidence was presented in favor of the idea that the physics of the fundamental string in six spacetime dimensions may equally well be described by a dual string and that one emerges as a soliton solution of the other [105, 134, 140, 146, 152, 155]. The string equations admits the singular *elementary* string solution [97]

$$ds^{2} = (1 - k^{2}/r^{2})[-d\tau^{2} + d\sigma^{2} + (1 - k^{2}/r^{2})^{-2}dr^{2} + r^{2}d\Omega_{3}^{2}]$$

$$e^{\Phi} = 1 - k^{2}/r^{2}$$

$$e^{-\Phi} * H_{3} = 2k^{2}\epsilon_{3}$$
(7.41)

where

$$k^2 = \kappa^2 T / \Omega_3 \tag{7.42}$$

 $T = 1/2\pi \alpha'$  is the string tension,  $\Omega_3$  is the volume of  $S^3$  and  $\epsilon_3$  is the volume form. It describes an infinitely long string whose worldsheet lies in the plane  $X^0 = \tau, X^1 = \sigma$ . Its mass per unit length is given by

$$M = T < e^{\Phi/2} >$$
 (7.43)

and is thus heavier for stronger string coupling, as one would expect for a fundamental string. The string equations also admit the non-singular *solitonic* string solution [134, 146]

$$ds^{2} = -d\tau^{2} + d\sigma^{2} + (1 - k^{2}/r^{2})^{-2}dr^{2} + r^{2}d\Omega_{3}^{2}$$
$$e^{-\Phi} = 1 - \tilde{k}^{2}/r^{2}$$
$$H_{3} = 2\tilde{k}^{2}\epsilon_{3}$$
(7.44)

whose tension  $\tilde{T} = 1/2\pi\tilde{\alpha}'$  is given by

$$\tilde{k}^2 = \kappa^2 \tilde{T} / \Omega_3 \tag{7.45}$$

Its mass per unit length is given by

$$\tilde{M} = \tilde{T} < e^{-\Phi/2} > \tag{7.46}$$

and is thus heavier for weaker string coupling, as one would expect for a solitonic string. Thus we see that the solitonic string differs from the fundamental string by the replacements

$$\Phi \to \Phi = -\Phi$$

$$G_{MN} \to \tilde{G}_{MN} = e^{-\Phi} G_{MN}$$

$$H \to \tilde{H} = e^{-\Phi} * H$$

$$\alpha' \to \tilde{\alpha}'$$
(7.47)

The Dirac quantization rule  $eg = 2\pi n$  (*n*=integer) relating the Noether "electric" charge

$$e = \frac{1}{\sqrt{2}\kappa} \int_{S^3} e^{-\Phi} * H_3 \tag{7.48}$$

to the topological "magnetic" charge

$$g = \frac{1}{\sqrt{2\kappa}} \int_{S^3} H_3 \tag{7.49}$$

translates into a quantization condition on the two tensions:

$$2\kappa^2 = n(2\pi)^3 \alpha' \tilde{\alpha}' \quad n = integer \tag{7.50}$$

where  $\kappa$  is the six-dimensional gravitational constant. Both the string and dual string soliton solutions break half the supersymmetries, both saturate a Bogomol'nyi bound between the mass and the charge. These solutions are the extreme mass equals charge limit of more general two-parameter black string solutions [107, 134].

We now make the major assumption of string/string duality: the dual string may be regarded as a fundamental string in its own right with a worldsheet action that couples to the dual variables and has the dual tension given in (7.47). It follows that the dual string equations admit the dual string (7.44) as the fundamental solution and the fundamental string (7.41) as the solution. When expressed in terms of the dual metric, however, the former is singular and the latter non-singular. It also follows from (7.50) that in going from the fundamental string to the dual string and interchanging  $\alpha'$  with  $\tilde{\alpha}' = 2\kappa^2/(2\pi)^3 \alpha'$ , one also interchanges the roles of worldsheet and spacetime loop expansions. Moreover, since the dilaton enters the dual string equations with the opposite sign to the fundamental string, it was argued in [105, 134, 146] that in D = 6 the strong coupling regime of the string should correspond to the weak coupling regime of the dual string:

$$g_6^2/(2\pi)^3 = \langle e^{\Phi} \rangle = (2\pi)^3/\tilde{g}_6^2$$
 (7.51)

where  $g_6$  and  $\tilde{g}_6$  are the six-dimensional string and dual string loop expansion parameters.

On compactification to four spacetime dimensions, the two theories appear very similar, each acquiring an O(2,2;Z) target space duality. One's first guess might be to assume that the strongly coupled four-dimensional fundamental string corresponds to the weakly coupled dual string, but in fact something more subtle and interesting happens: the roles of the S and T fields are interchanged [152] so that the strong/weak coupling  $SL(2,Z)_S$  of the fundamental string emerges as a subgroup of the target space duality of the dual string:

$$O(2,2;Z)_{SU} \sim SL(2,Z)_S \times SL(2,Z)_U$$
 (7.52)

This duality of dualities is summarized in Table 7.4. As a consistency check, we note that since  $(2\pi R)^2/2\kappa^2 = 1/16\pi G$  the Dirac quantization rule (7.50) becomes (choosing n=1)

$$8GR^2 = \alpha'\tilde{\alpha}' \tag{7.53}$$

Invariance of this rule now requires that a strong/weak coupling transformation on the fundamental string  $(8G/\alpha' \rightarrow \alpha'/8G)$  must be accompanied by a minimum/maximum length transformation of the dual string  $(\tilde{\alpha}'/R^2 \rightarrow R^2/\tilde{\alpha}')$ , and vice versa.

The idea of a D = 4 S-duality arising as a T-duality in going from D = 6 on  $T^2$ later appeared in the Langlands programme in pure mathematics [309]. A similar construction is the D = 10 S-duality of Type IIB arising as a T-duality in going from D = 12 on  $T^2$  in F-theory [187].

• Four-dimensional Heterotic/Type IIA/Type IIB triality

T-duality	$O(2,2;Z)_{TU}$	$O(2,2;Z)_{SU}$
	$\sim SL(2,Z)_T \times SL(2,Z)_U$	$\sim SL(2,Z)_S \times SL(2,Z)_U$
Moduli	$T = \mathbf{b} + i e^{-\sigma}$	$S = \mathbf{a} + i e^{-\eta}$
	$b = B_{45}$	$\mathbf{a} = \tilde{B}_{45}$
	$e^{-\sigma} = \sqrt{detG_{mn}}$	$e^{-\eta} = \sqrt{det\tilde{G}_{mn}}$
Worldsheet coupling	$< e^{\sigma} >= \alpha'/R^2$	$< e^{\eta} >= g^2/2\pi$
Large/smallradius	$R \to \alpha'/R$	$g^2/2\pi \to 2\pi/g^2$
S-duality	$SL(2,Z)_S$	$SL(2,Z)_T$
Axion/dilaton	$S = \mathbf{a} + i e^{-\eta}$	$T = \mathbf{b} + i e^{-\sigma}$
	$d\mathbf{a} = e^{-\eta} \ast H$	$d\mathbf{b} = e^{-\sigma} \tilde{*} \tilde{H}$
	$e^{-\eta} = e^{-\Phi} \sqrt{det G_{mn}}$	$e^{-\sigma} = e^{\Phi} \sqrt{det \tilde{G}_{mn}}$
$Spacetime \ coupling$	$< e^{\eta} >= g^2/2\pi$	$< e^{\sigma} >= \alpha'/R^2$
Strong/weak coupling	$g^2/2\pi \to 2\pi/g^2$	$R \to \alpha'/R$

 $Fundamental\ string$ 

 $Dual\,string$ 

Table 8: Duality of dualities

We have seen that in six spacetime dimensions, the heterotic string is dual to a Type IIA string. On further toroidal compactification to four spacetime dimensions, the heterotic string acquires an  $SL(2, Z)_S$  strong/weak coupling duality and an  $SL(2,Z)_T \times SL(2,Z)_U$  target space duality acting on the dilaton/axion, complex Kahler form and the complex structure fields S, T, U respectively. Strong/weak duality in D = 6 interchanges the roles of S and T in D = 4yielding a Type IIA string with fields T, S, U. However, the target space symmetry of the heterotic theory also contains an  $SL(2,Z)_U$  that acts on U, the complex structure of the torus. This suggests that, in addition to these S and T strings there ought to be a third U-string whose axion/dilaton field is U and whose strong/weak coupling duality is  $SL(2, Z)_U$ . From a D = 6 perspective, this seems strange since we now interchange  $G_{45}$  and  $B_{45}$ . Moreover, of the two electric field strengths which become magnetic, one is a winding gauge field and the other is Kaluza-Klein! So such a duality has no D = 6 Lorentz invariant meaning. In fact, this U string is a Type IIB string, a result which may also be understood from the point of view of mirror symmetry: interchanging the roles of Kahler form and complex structure (which is equivalent to inverting the radius of one of the two circles) is a symmetry of the heterotic string but takes Type IIA into Type IIB [85, 88]. In summary, if we denote the heterotic, IIA and IIB strings by H, A, B respectively and the axion/dilaton, complex Kahler form and complex structure by the triple XYZ then we have a triality between the S-string  $(H_{STU} = H_{SUT})$ , the T-string  $(B_{TUS} = A_{TSU})$  and the U-string  $(A_{UST} = B_{UTS})$ . We note that D = 6 general covariance is a perturbative symmetry of the Type IIB string and therefore that the D = 4 Type IIB strings must have a perturbative SL(2, Z) acting on the complex structure of the compactifying torus. Secondly we note that for both Type IIB theories,  $B_{TUS}$  and  $B_{UTS}$ , S is the complex structure field. Thus the T string has  $SL(2,Z)_U \times SL(2,Z)_S$  and the U string has  $SL(2,Z)_S \times SL(2,Z)_T$  as required. In this sense, four-dimensional string/string/string triality fills a gap left by six-dimensional string/string duality: although duality satisfactorily explains the strong/weak coupling duality of the D = 4 Type IIA string in terms of the target space duality of the heterotic string, the converse requires the Type IIB ingredient [166].

• Triality and the STU model

An interesting subsector of string compactification to four dimensions is provided



Figure 9: Heterotic/Type IIA/Type IIB triality. The solid lines correspond to string/string dualities and the dashed lines represent mirror transformations.

by the STU model. This model has a low energy limit which is described by N = 2 supergravity coupled to three vector multiplets interacting through the special Kahler manifold  $[SL(2)/SO(2)]^3$ . The three complex scalars are denoted by the letters S, T and U, hence the name of the model [166, 167]. The remarkable feature that distinguishes it from generic N=2 supergravities is its S-T-U triality. A general static spherically symmetric black hole solution depends on 4 electric and 4 magnetic charges denoted  $q_0, q_1, q_2, q_3, p^0, p^1, p^2, p^3$ , but the generating solution depends on just 8-3 = 5 parameters. The solution can usefully be embedded in N = 4 supergravity with symmetry  $SL(2) \times SO(6, 22)$ , the low-energy limit of the heterotic string compactified on  $T^6$ , where the charges transform as a (2, 28)and also in N = 8 supergravity with symmetry  $E_{7(7)}$ , the low-energy limit of the Type IIA or Type IIB strings, compactified on  $T^6$  or M-theory on  $T^7$ , where the charges transform as a 56. In both cases, remarkably, the same five parameters suffice to describe these 56-charge black holes [182, 203] after fixing the action of the isotropy subgroup  $[SO(2)]^3$ . The STU black hole entropy is a complicated function of the 8 charges [167]:

$$(S/\pi)^2 = -(p \cdot q)^2 + 4 \Big[ (p^1 q_1)(p^2 q_2) + (p^1 q_1)(p^3 q_3) + (p^3 q_3)(p^2 q_2) + q_0 p^1 p^2 p^2 - p^0 q_1 q_2 q_3 \Big]$$

$$(7.54)$$

Some examples of supersymmetric black hole solutions [141] are provided by the electric Kaluza-Klein black hole with q = (1, 0, 0, 0) and p = (0, 0, 0, 0); the magnetic Kaluza-Klein black hole with q = (0, 0, 0, -1) and p = (0, -1, 0, 0); the magnetic Kaluza-Klein black hole with q = (0, 0, 0, 0) and p = (0, -1, 0, 0); the magnetic winding black hole with q = (0, 0, 0, 0) and p = (0, 0, -1, 0, 0); the magnetic winding black hole with q = (1, 0, 0, 0) and p = (0, 0, -1, 0, 0); the magnetic for the 4-particle example, however, which is just the Reissner-Nordstrom black hole.

# 8 1996 Ten to eleven: It is not too late

INTERNATIONAL SCHOOL OF SUBNUCLEAR PHYSICS - Director: A. ZICHICHI
34th Course: Effective Theories and Fundamental Interactions - Directors: G. VENEZIANO
- A. ZICHICHI 3 - 12 July 1996

### 8.1 M-theory: the theory formerly known as strings

Not so long ago it was widely believed that there were five different superstring theories each competing for the title of "Theory of everything," that all-embracing theory that describes all physical phenomena. See Table 9.

Moreover, on the (d, D) brane-scan of supersymmetric extended objects with d worldvolume dimensions moving in a spacetime of D dimensions, all these theories occupied the same (d = 2, D = 10) slot. See Table 1. The orthodox wisdom was that while (d = 2, D = 10) was the Theory of Everything, the other branes on the scan were Theories of Nothing. All that has now changed. We now know that there are not five different theories at all but, together with D = 11 supergravity, they form merely six different corners of a deeper, unique and more profound theory called "M-theory" where M stand for Magic, Mystery or Membrane. M-theory involves all of the other branes on the brane-scan, in particular the eleven-dimensional membrane (d = 3, D = 11) and eleven-dimensional fivebrane (d = 6, D = 11), thus resolving the mystery of why strings stop at ten dimensions while supersymmetry allows eleven [54].

Although we can glimpse various corners of M-theory, the big picture still eludes us. Uncompactified M-theory has no dimensionless parameters, which is good from

	Gauge Group	Chiral?	Supersymmetry charges
Type I	SO(32)	yes	16
Type IIA	U(1)	no	32
Type IIB	_	yes	32
Heterotic	$E_8 \times E_8$	yes	16
Heterotic	SO(32)	yes	16

Table 9: The Five Superstring Theories

the uniqueness point of view but makes ordinary perturbation theory impossible since there are no small coupling constants to provide the expansion parameters. A low energy, E, expansion is possible in powers of  $E/M_P$ , with  $M_P$  the Planck mass, and leads to the familiar D = 11 supergravity plus corrections of higher powers in the curvature. Figuring out what governs these corrections would go a long way in pinning down what M-theory really is.

Why, therefore, do we place so much trust in a theory we cannot even define? First we know that its equations (though not in general its vacua) have the maximal number of 32 supersymmetry charges. This is a powerful constraint and provides many "What else can it be?" arguments in guessing what the theory looks like when compactified to D < 11 dimensions. For example, when *M*-theory is compactified on a circle  $S^1$  of radius  $R_{11}$ , it leads to the Type IIA string, with string coupling constant  $g_s$  given by

$$g_s = R_{11}^{3/2} \tag{8.1}$$

We recover the weak coupling regime only when  $R_{11} \rightarrow 0$ , which explains the earlier illusion that the theory is defined in D = 10. Similarly, if we compactify on a line segment  $S^1/Z_2$ , we recover the  $E_8 \times E_8$  heterotic string. Moreover, although the corners of M-theory we understand best correspond to the weakly coupled, perturbative, regimes where the theory can be approximated by a string theory, they are related to one another by a web of dualities, some of which are rigorously established and some of which are still conjectural but eminently plausible. For example, if we further compactify Type IIA string on a circle of radius R, we can show rigorously that it is equivalent to the Type IIB string compactified on a circle or radius 1/R. If we do the same thing for the  $E_8 \times E_8$  heterotic string we recover the SO(32) heterotic string. These well-established relationships which remain within the weak coupling regimes are called *T*-dualities. The name *S*-dualities refers to the less well-established strong/weak coupling relationships. For example, the SO(32) heterotic string is believed to be Sdual to the SO(32) Type I string, and the Type IIB string to be self-S-dual. If we compactify more dimensions, other dualities can appear. For example, the heterotic string compactified on a six-dimensional torus  $T^6$  is also believed to be self-S-dual. There is also the phenomenon of duality of dualities by which the T-duality of one theory is the S-duality of another. When M-theory is compactified on  $T^n$ , these S and T dualities are combined into what are termed U-dualities. All the consistency checks we have been able to think of (and after 20+ years there are dozens of them) have worked and convinced us that all these dualities are in fact valid. Of course we can compactify M-theory on more complicated manifolds such as the four-dimensional K3 or the six-dimensional Calabi-Yau spaces and these lead to a bewildering array of other dualities. For example: the heterotic string on  $T^4$  is dual to the Type II string on K3; the heterotic string on  $T^6$  is dual to the the Type II string on Calabi-Yau; the Type IIA string on a Calabi-Yau manifold is dual to the Type IIB string on the mirror Calabi-Yau manifold. These more complicated compactifications lead to many more parameters in the theory, known to the mathematicians as *moduli*, but in physical uncompactified spacetime have the interpretation as expectation values of scalar fields [25]. Within string perturbation theory, these scalar fields have flat potentials and their expectation values are arbitrary. So deciding which topology Nature actually chooses and the values of the moduli within that topology is known as the vacuum degeneracy problem.

# 8.2 String/string duality from M-theory

Let us consider M-theory, with its fundamental membrane and solitonic fivebrane, on  $R^6 \times M^1 \times \tilde{M}^4$  where  $M^1$  is a one-dimensional compact space of radius R and  $\tilde{M}^4$  is a four-dimensional compact space of volume V. We may obtain a fundamental string on  $R^6$  by wrapping the membrane around  $M^1$  and reducing on  $\tilde{M}^4$ . Let us denote the fundamental string sigma-model metrics in D = 10 and D = 6 by  $G_{10}$  and  $G_6$ . Then from the corresponding Einstein Lagrangians

$$\sqrt{-G_{11}}R_{11} = R^3\sqrt{-G_{10}}R_{10} = \frac{V}{R}\sqrt{-G_6}R_6 \tag{8.2}$$

we may read off the strength of the string couplings in D = 10 [185]

$$\lambda_{10}^2 = R^3 \tag{8.3}$$

and D = 6

$$\lambda_6^2 = \frac{R}{V} \tag{8.4}$$

Similarly we may obtain a solitonic string on  $R^6$  by wrapping the fivebrane around  $\tilde{M}^4$  and reducing on  $M^1$ . Let us denote the solitonic string sigma-model metrics in D = 7 and D = 6 by  $\tilde{G}_7$  and  $\tilde{G}_6$ . Then from the corresponding Einstein Lagrangians

$$\sqrt{-G_{11}}R_{11} = V^{-3/2}\sqrt{-\tilde{G}_7}\tilde{R}_7 = \frac{R}{V}\sqrt{-\tilde{G}_6}\tilde{R}_6$$
(8.5)

we may read off the strength of the string couplings in D = 7 [185]

$$\tilde{\lambda}_7{}^2 = V^{-3/2} \tag{8.6}$$

and D = 6

$$\tilde{\lambda}_6{}^2 = \frac{V}{R} \tag{8.7}$$

Thus we see that the fundamental and solitonic strings are related by a strong/weak coupling:

$$\tilde{\lambda}_6^2 = 1/{\lambda_6}^2 \tag{8.8}$$

We shall be interested in  $M^1 = S^1$  (in which case the fundamental string will be Type IIA) or  $M^1 = S^1/Z^2$  (in which case the fundamental string will be heterotic  $E_8 \times E_8$ ). Similarly, we will be interested in  $\tilde{M}^4 = T^4$  (in which case the solitonic string will be Type IIA) or  $\tilde{M}^4 = K3$  (in which case the solitonic string will be heterotic). Thus there are four possible scenarios which are summarized in Table 10.  $(N_+, N_-)$ denote the D = 6 spacetime supersymmetries. In each case, the fundamental string will be weakly coupled as we shrink the size of the wrapping space  $M^1$  and the dual string will be weakly coupled as we shrink the size of the wrapping space  $\tilde{M}^4$ .

In fact, there is in general a topological obstruction to wrapping the fivebrane around  $\tilde{M}^4$  provided by

$$\int F_4 = 2\pi m \tag{8.9}$$

where F is the 4-form field strength of D = 11 supergravity, because the fivebrane cannot wrap around a 4-manifold that has  $m \neq 0$ . This is because the anti-self-dual 3-form field strength T on the worldvolume of the fivebrane obeys

$$dA_3 = F_4 \tag{8.10}$$

$(N_{+}, N_{-})$	$M^1$	$\tilde{M}^4$	Fundamental string	Dual string
------------------	-------	---------------	--------------------	-------------

$(1,\!0)$	$S^{1}/Z_{2}$	$\mathbf{K3}$	heterotic	heterotic
$(1,\!1)$	$S^1$	K3	Type IIA	heterotic
$(1,\!1)$	$S^1/Z_2$	$T^4$	heterotic	Type IIA
$(2,\!2)$	$S^1$	$T^4$	Type IIA	Type IIA

Table 10: String/String dualities

and the existence of a solution for  $A_3$  therefore requires that  $F_4$  must be cohomologically trivial. For M-theory on  $R^6 \times S^1/Z^2 \times T^4$  this is no problem. However, for M-theory on  $R^6 \times S^1/Z^2 \times K_3$ , with instanton number k in one  $E_8$  and (24 - k) in the other, the flux of  $F_4$  over K3 is [185]

$$m = 12 - k \tag{8.11}$$

Consequently, the M-theoretic explanation of heterotic/heterotic duality requires  $E_8 \times E_8$  with the symmetric embedding k = 12. This has some far-reaching implications. For example, the duality exchanges gauge fields that can be seen in perturbation theory with gauge fields of a nonperturbative origin [185].

The dilaton  $\tilde{\Phi}$ , the string sigma-model metric  $\tilde{G}_{MN}$  and 3-form field strength  $\tilde{H}$  of the dual string are related to those of the fundamental string,  $\Phi$ ,  $G_{MN}$ , and H by the replacements

$$\Phi \to \Phi = -\Phi$$

$$G_{MN} \to \tilde{G}_{MN} = e^{-\Phi} G_{MN}$$

$$H \to \tilde{H} = e^{-\Phi} * H$$
(8.12)

In the case of heterotic/Type IIA duality and Type IIA/heterotic duality, this operation takes us from one string to the other, but in the case of heterotic/heterotic duality and Type IIA/Type IIA duality this operation is a discrete symmetry of the theory. This Type IIA/Type IIA duality is hardly ever discussed in the literature in these terms, but we can recognise this symmetry as a subgroup of the SO(5, 5; Z) U-duality of the D = 6 Type IIA string.

### 8.3 Subsequent developments

• These include F-theory [187], strong coupling expansion of Calabi-Yau compactifications [188], and string dynamics in six dimensions [190, 191, 202].

# 9 2003 The status of local supersymmetry

INTERNATIONAL SCHOOL OF SUBNUCLEAR PHYSICS - Director: A. ZICHICHI 41st Course: From Quarks to Black Holes: Progress in Understanding the Logic of Nature Directors: G. t HOOFT - A. ZICHICHI 29 August - 7 September 2003

# 9.1 Supersymmetry without Supermembranes: Not an option

Gravity exists, so if there is any truth to supersymmetry then any realistic supersymmetry theory must eventually be enlarged to a supersymmetric theory of matter and gravitation, known as supergravity. Supersymmetry without supergravity is not an option, though it may be a good approximation at energies below the Planck Scale.

Steven Weinberg, The Quantum Theory of Fields, Volume III, Supersymmetry To paraphrase Weinberg:

Supergravity is itself only an effective nonrenormalizable theory which breaks down at the Planck energies. So if there is any truth to supersymmetry then any realistic theory must eventually be enlarged to superstrings which are ultraviolet finite. Supersymmetry without superstrings is not an option.

To paraphrase Weinberg again:

Superstring theory is itself only a perturbative theory which breaks down at strong coupling. So if there is any truth to supersymmetry then any realistic theory must eventually be enlarged to the non-perturbative M-theory, a theory involving higher dimensional extended objects: the super p-branes<sup>13</sup>. Supersymmetry without M-theory is not an option.

Yet two of the most basic questions of M-theory have until now remained unanswered:

<sup>&</sup>lt;sup>13</sup>In my opinion, calling M-theory the strong coupling limit of string theory is a bit like calling string theory the high-energy limit of general relativity.
d/(11 - d)	G(spacelike)	G(null)	G(timelike)
11/0	{1}	{1}	{1}
10/1	{1}	{1}	{1}
9/2	SO(2)	R	$\mathrm{SO}(1,1)$
8/3	$SO(3) \times SO(2)$	$ISO(2) \times R$	$SO(2,1) \times SO(1,1)$
7/4	SO(5)	$[\mathrm{SO}(3)\times\mathrm{SO}(2)]\times\mathrm{R6}_{(3,2)}$	$\mathrm{SO}(3,2)$
6/5	$SO(5) \times SO(5)$	$SO(5) \times R^{10}_{(10)}$	SO(5, C)
5/6	USp(8)	$[\mathrm{SO}(5) \times \mathrm{SO}(5)] \times \mathrm{R}^{16}_{(4,4)}$	$\mathrm{USp}(4,4)$
4/7	SU(8)	$\mathrm{USp}(8) \times \mathrm{R}^{27}_{(27)}$	$\mathrm{SU}^*(8)$
3/8	SO(16)	$[\mathrm{SU}(8) \times \Upsilon(1)] \times \mathrm{R}^{56}_{(28_{1/2},\overline{28}_{-1/2})}$	$\mathrm{SO}^*(16)$
2/9	$SO(16) \times SO(16)$	$SO(16) \times R^{120}_{(120)}$	SO(16, C)
1/10	SO(32)	$[SO(16) \times SO(16)] \times R^{256}_{(16,16)}$	$\mathrm{SO}(16, 16)$
0/11	SL(32, R)	$\mathrm{SL}(32,\mathrm{R})$	SL(32, R)

Table 11: The generalized structure groups are given by  $\mathcal{G} = \mathrm{SO}(d-1,1) \times G(spacelike)$ ,  $\mathcal{G} = \mathrm{ISO}(d-1) \times G(null)$  and  $\mathcal{G} = \mathrm{SO}(d) \times G(timelike)$ .

# 9.2 What are the D=11 symmetries?

In this lecture we argued that the equations of M-theory possess previously unidentified hidden spacetime (timelike and null) symmetries in addition to the well-known hidden internal (spacelike) symmetries. For  $11 \ge d \ge 3$ , these coincide with the generalized structure groups discussed below and take the form  $\mathcal{G} = SO(d-1,1) \times G(spacelike)$ ,  $\mathcal{G} = ISO(d-1) \times G(null)$  and  $\mathcal{G} = SO(d) \times G(timelike)$  with  $1 \le d < 11$ . For example,  $G(spacelike) = SO(16), G(null) = [SU(8) \times U(1)] \times R^{56}$  and  $G(timelike) = SO^*(16)$ when d = 3. The nomenclature derives from the fact that these symmetries also show up in the spacelike, null and timelike dimensional reductions of the theory. However, we emphasize that we are proposing them as background-independent symmetries of the full unreduced and untruncated D = 11 equations of motion, not merely their dimensional reduction. Although extending spacetime symmetries, there is no conflict with the Coleman-Mandula theorem. A more speculative idea [273] is that there exists a yet-to-be-discovered version of D = 11 supergravity or M-theory that displays even bigger hidden symmetries corresponding to  $\mathcal{G}$  with  $d \le 3$  which could be as large as SL(32, R).

#### 9.3 Counting supersymmetries of M-theory vacua

The equations of M-theory display the maximum number of supersymmetries N=32, and so n, the number of supersymmetries preserved by a particular vacuum, must be some integer between 0 and 32. But are some values of n forbidden and, if so, which ones? For quite some time it was widely believed that, aside from the maximal n = 32, n is restricted to  $0 \le n \le 16$  with n = 16 being realized by the fundamental BPS objects of M-theory: the M2-brane, the M5-brane, the M-wave and the M-monopole. The subsequent discovery of intersecting brane configurations with n = 0, 1, 2, 3, 4, 5, 6, 8, 16 lent credence to this argument. On the other hand, it has been shown that all values  $0 \le n \le 32$  are in principle allowed by the M-theory algebra and examples of vacua with 16 < n < 32 have indeed since been found.

In M-theory vacua with vanishing 4-form  $F_{(4)}$ , one can invoke the ordinary Riemannian holonomy  $H \subset SO(10, 1)$  to account for unbroken supersymmetries n =1, 2, 3, 4, 6, 8, 16, 32. To explain the more exotic fractions of supersymmetry, in particular 16 < n < 32, we need to generalize the notion of holonomy to accommodate non-zero  $F_{(4)}$ . We show that the number of supersymmetries preserved by an Mtheory vacuum is given by the number of singlets appearing in the decomposition of the 32-dimensional representation of  $\mathcal{G}$  under  $\mathcal{G} \supset \mathcal{H}$  where  $\mathcal{G}$  are generalized structure groups that replace SO(1, 10) and  $\mathcal{H}$  are generalized holonomy groups. In general we require the maximal  $\mathcal{G}$ , namely SL(32, R), but smaller  $\mathcal{G}$  appear in special cases such as product manifolds.

#### 9.4 Subsequent developments

• Generalized holonomy

Generalized holonomy is developed further in [272, 273, 282, 340]. We conjectured, albeit on flimsy evidence, that the number of vacuum supersymmetries allowed by M-theory is restricted to

n=0,1,2,3,4,5,6,8,10,12,14,16,18,20,22,24,26,28,32

Interestingly enough, a Godel universe with n = 14 was subsequently discovered [270] which completes this list. Furthermore: n = 31 has now been ruled out for both Type IIB [291] and Type IIA [292]. n = 30 has now been ruled out for M-theory [312]. The class of M-theory plane waves found in [312] has = 16, 20, 26

but not n = 28, although plane wave solutions with n = 28 do appear in Type IIB [266]. Backgrounds with n > 24 are necessarily (locally) homogeneous. See [321] where it is also conjectured that 24 is the minimum number which guarantees this. See [372] for a recent review.

• LHC

According to much of the popular media (and even some phenomenologists and experimentalists), the failure to find supersymmetric particles at the LHC signals the demise of supersymmetry. See, for example,

https://www.economist.com/science-and-technology/2016/11/12/a-bet-about-a-cherished-theory-of-physics-may-soon-pay-out

https://www.forbes.com/sites/startswithabang/2017/10/06/five-brilliant-ideas-for-new-physics-that-need-to-die-already/#4472764857b7

https://www.scientificamerican.com/article/is-supersymmetry-dead/

https://www.theguardian.com/science/2013/jun/16/has-physics-gone-too-far

Since the super in superstring and supermembrane refers to supersymmetry, this failure to detect any superpartners is also said to cast doubt on string and M-theory, but string and M-theory are compatible with supersymmetry becoming evident only at much higher energies. In fact, they are silent about what energies supersymmetry would reveal itself. This is a valid criticism of our current state of knowledge but it is not a "fudge", as some journalists have claimed. One would expect to see super-particles at the LHC only if supersymmetry is the solution to the "gauge-hierarchy problem". This is an extra assumption, favoured by some particle physicists, but it is not intrinsic to supersymmetry. Many of those same journalists think that supersymmetry was invented to solve the gauge hierarchy problem when in fact it precedes it.

When in the 1970s, encouraged by Abdus Salam and Chris Isham at Imperial College, I embarked on a career devoted to quantum gravity (a force forty orders of magnitude weaker than the others) I was well aware that this meant a departure from the kind of close association of theory and experiment traditionally enjoyed by particle physicists. We were in it for the long haul and empirical confirmation, if it came at all, was likely to be indirect. Nevertheless I thought it worthwhile given that the unification of gravity and quantum theory is the most important unresolved quandary in science. Strange then that many journalists seem to regard this as new problem unique to string/M theory and/or supersymmetry. I, along with many others belonging to the "hep-th" wing of theoretical physics, was attracted to global and local supersymmetry because they are respectively the square root of special and general relativity and hence provide a natural framework for incorporating gravity. There were three outstanding issues in quantum gravity in the 1970s: (1) Ultraviolet divergences and non-renormalizabilIty (2) The microscopic origin of the Bekenstein-Hawking black hole entropy (3) The black hole information paradox. Supersymmetry in the form of string theory has since provided an answer to (1), supersymmetry in the form of M-theory has (according to Hawking and others) since provided an answer to (3). Moreover, I know of no other theory that provides adequate answers to any of (1), (2) or (3). Supersymmetry is still alive and kicking.

# 10 2010 Black holes, qubits and quantum information

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#### 10.1 Three qubits: Alice, Bob and Charlie

The three qubit system (where A, B, C = 0, 1) is described by the state

$$|\Psi\rangle = a_{ABC} |ABC\rangle$$
  
=  $a_{000}|000\rangle + a_{001}|001\rangle + a_{010}|010\rangle + a_{011}|011\rangle$   
+ $a_{100}|100\rangle + a_{101}|101\rangle + a_{110}|110\rangle + a_{111}|111\rangle$  (10.1)

The tripartite entanglement of Alice, Bob and Charlie is given by the "3-tangle" [243]

$$\tau_{ABC} = 4|\text{Det } a_{ABC}| \tag{10.2}$$

here Det  $a_{ABC}$  is Cayley's hyperdeterminant quartic in the hypermatrix  $a_{ABC}$ 

Det 
$$a_{ABC} = -\frac{1}{2} \epsilon^{A_1 A_2} \epsilon^{B_1 B_2} \epsilon^{A_3 A_4} \epsilon^{B_3 B_4} \epsilon^{C_1 C_4} \epsilon^{C_2 C_3} a_{A_1 B_1 C_1} a_{A_2 B_2 C_2} a_{A_3 B_3 C_3} a_{A_4 B_4 C_4}$$

$$=a_{000}^2a_{111}^2 + a_{001}^2a_{110}^2 + a_{010}^2a_{101}^2 + a_{100}^2a_{011}^2$$
(10.3)

 $-2(a_{000}a_{001}a_{110}a_{111} + a_{000}a_{010}a_{101}a_{111} + a_{000}a_{100}a_{011}a_{111} + a_{001}a_{010}a_{101}a_{101}a_{110}$ 

 $+a_{001}a_{100}a_{011}a_{110} + a_{010}a_{100}a_{011}a_{101}) + 4(a_{000}a_{011}a_{101}a_{110} + a_{001}a_{010}a_{100}a_{111})$  (10.4) The hyperdeterminant is invariant under  $SL(2)_A \times SL(2)_B \times SL(2)_C$ , with  $a_{ABC}$  transforming as a (2, 2, 2), and under a discrete triality that interchanges A, B and C.

# 10.2 STU black holes

Remarkably, there is a correspondence between this measure of tripartite entanglement of three qubits and the entropy S of the 8-charge STU black hole of Section 7.4. By identifying the 8 charges with the 8 components of the three-qubit hypermatrix  $a_{ABC}$ ,

$$(p^0, p^1, p^2, p^3, q_0, q_1, q_2, q_3) = (a_{000}, -a_{001}, -a_{111}, -a_{011}, a_{110}, a_{101}, a_{101}, a_{010})$$
(10.5)

one finds

$$S = \pi \sqrt{|\text{Det } a_{ABC}|} = \frac{\pi}{2} \sqrt{\tau_{ABC}}$$
(10.6)

This turns out to be just the tip of an iceberg and further papers [285, 287, 293, 294, 295, 298, 307, 300, 325] have written a more complete dictionary, which translates a variety of phenomena in one language to those in the other. For example, in the N = 2 theory the 3-qubit entanglement classification, is matched by the black hole classification with either 1/2 or 0 fraction of supersymmetry preserved. By embedding in the N = 8 theory, we can include the finer supersymmetry preserving distinctions.

There is, in fact, a quantum information theoretic interpretation of the 56 charge N = 8 black hole in terms of a Hilbert space consisting of 7 copies of the 3-qubit Hilbert space [293, 301]. It relies on  $[SL(2)]^7$  being a subgroup of  $E_{7(7)}$  and admits the interpretation, via the Fano plane, of a tripartite entanglement of seven qubits, with the entanglement measure given by Cartans quartic  $E_{7(7)}$  invariant. Remarkably, however, because the generating solution depends on the same 5 parameters as the STU model, its classification of states will exactly parallel that of the usual 3-qubits. Indeed, the Cartan invariant reduces to Cayleys hyperdeterminant in a canonical basis [285]. Nevertheless, we still do not know whether there are any physical reasons underlying these mathematical coincidences.

4	1	5	6	7	8	9	macro charges	micro charges	ABC>
2	x	0	x	0	x	0	$p^0$	0	000 >
	C	x	0	х	x	0	$q_1$	0	110 >
	C	x	х	0	0	x	$q_2$	$-N_3 \sin\theta\cos\theta$	101 >
2	ĸ	0	0	Х	0	x	$q_3$	$N_3 { m sin}  heta { m cos}  heta$	011 >
0	C	x	0	х	0	x	$q_0$	$N_0 + N_3 \sin^2 \theta$	111 >
2	ĸ	0	x	0	0	x	$-p^1$	$-N_3 \cos^2 \theta$	001 >
2	ĸ	0	0	X	x	0	$-p^2$	$-N_2$	010 >
0	C	х	х	0	x	0	$-p^3$	$-N_1$	100 >

Table 12: Three qubit interpretation of the 8-charge D=4 black hole from four D3-branes wrapping around the lower four cycles of  $T^6$  with wrapping numbers  $N_0, N_1, N_2, N_3$ . Note that they intersect over a string at angle  $\theta$ .

# 10.3 Wrapped branes as qubits

In the same spirit we consider the configurations of intersecting D3-branes, whose wrapping around the six compact dimensions  $T^6$  provides the microscopic string-theoretic interpretation of the charges, and associate the three-qubit basis vectors  $|ABC\rangle$ , (A, B, C = 0 or 1) with the corresponding 8 wrapping cycles.

Thus our microscopic analysis of the black hole has provided an explanation for the appearance of the qubit two-valuedness (0 or 1) that was lacking in the previous macroscopic treatments [284, 285, 287, 293, 294, 298, 295]: the brane can wrap one circle or the other in each  $T^2$ . The number of qubits is three because of the six extra dimensions of string theory.

To wrap or not to wrap: that is the qubit.

In particular, we relate a well-known fact of quantum information theory, that the most general real three-qubit state can be parameterized by four real numbers and an angle, to a well-known fact of string theory, that the most general STU black hole can be described by four D3-branes intersecting at an angle.

# 10.4 Subsequent developments

• Quantum information

Falsifiable predictions in the fields of high-energy physics or cosmology are hard to come by, especially for ambitious attempts, such as string/M-theory, to accommodate all the fundamental interactions. In the field of quantum information theory, however, the work described in this lecture has shown that the stringy black hole/qubit correspondence can reproduce well-known results in the classification of two and three qubit entanglement. In [314] this correspondence was taken one step further to predict new results in the less well-understood case of four-qubit entanglement that can in principle be tested in the laboratory.

• It from bit?

Looking at the hep-th arXiv in 2018, we see that quantum information has become a dominant theme that has attracted the attention of leading string theorists, for example [355, 359, 371]. However, we cannot claim much credit for this since these developments have not followed the kind of black hole/qubit correspondence discussed above. A different kind of black hole/qubit correspondence, namely ER=EPR has been very influential [331] as has the holographic derivation of entanglement entropy [286, 288].

# 11 2016 M-physics

# 11.1 Oxford English Dictionary: M-theory

#### M-theory, n. Particle Physics.

Brit.  $\epsilon m \theta ieri$ , U.S.  $\epsilon m \theta ieri, \, \epsilon m \theta eri$ 

[; M (app. representing MEMBRANE n.) + THEORY n.1]

A unified theory involving branes that subsumes eleven-dimensional supergravity and the five ten-dimensional superstring theories.

Quot. 1996 is from a paper received for publication earlier (23 Oct. 1995) than quot. 1995 (17 Dec.).

1995 Re: Confinement: Massive Gauge Bosons in sci.physics (Usenet newsgroup) 17 Dec., String theorists are a mathematically sophisticated crew, so I'm sure they would enjoy an abstract description of the M-theory (as it's called) from which one could then derive all its varied guises.

**1996** J. H. Schwarz in Physics Lett. B. 367 97/1 If one assumes the existence of a fundamental theory in eleven dimensions (let's call it the M theory), this provides a powerful heuristic basis for understanding various results in string theory. [Note] This name was suggested by E. Witten.

**1998** Sci. Amer. Feb. 59/2 Despite all these successes, physicists are glimpsing only small corners of M-theory; the big picture is still lacking.

**2002** U.S. News & World Rep. 6 May 59/1 M-theory..holds that our universe may occupy just part of a many-dimensional mega-universe. In that picture, it could be shadowed by another universe on a different brane- M-theory jargon for 3-D membrane.

# 11.2 Where M stands for...

More M-etymology:

"Recent results indicate that if one assumes the existence of a fundamental theory in eleven dimensions (let's call it the 'M-theory' [This name was suggested by E. Witten]), this provides a powerful heuristic basis for understanding various results in string theory." J. Schwarz, hep-th/9510101.

"As it has been proposed that the eleven-dimensional theory is a supermembrane theory but there are some reasons to doubt that interpretation, we will non-committally call it M-theory, leaving for future the relation of M to membranes." P. Horava and E. Witten, hep-th/9510209

"For instance, the eleven-dimensional 'M-theory' (where M stands for magic, mystery or membrane, according to taste) on  $X \times S^1$ , with X any ten-manifold, is equivalent to Type IIA on X, with a Type IIA string coupling constant that becomes small when the radius of  $S^1$  goes to zero." E. Witten, hep-th/9512219

## 11.3 Subsequent developments

• M-theory and string theory

What is the future of branes? I will finish on an optimistic note borrowed from Scientific American [228] (and Isaac Newton):

Edward Witten is fond of imagining how physics might develop on another planet, where major discoveries such as general relativity, quantum mechanics and supersymmetry are made in a different order than on Earth. In a similar vein, I would like to suggest that on planets more logical than ours, branes in 11 dimensions would have been the starting point from which 10-dimensional string theory was subsequently derived. Indeed, future terrestrial historians may judge the late 20th century as a time when theorists were like children playing on the seashore, diverting themselves in now and then finding a smoother pebble or prettier shell in superstrings, whilst the great ocean of M-theory lay all undiscovered before them.

# 12 2017 Thirty years of Erice on the Brane

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# 12.1 Nino

See Figure 10.

# 12.2 Subsequent developments

This paper.

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Figure 10: Nino and the author

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